有限群的块

块的超聚焦子代数

路易斯 步驰

LLUIS PUIG

Blocks of finite Groups

The Hyperfocal subalgebra of a Block

...two indecomposable parts U_κ and U_λ of R belong to the same block if there is a sequence of U_ρ of the form

$$(32) \quad U_\kappa,\ U_\sigma, \ldots\ldots,\ U_\tau,\ U_\lambda$$

such that any two neighbouring U_ρ in (32) have at least one irreducible constituent in common...

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1. 引言

1.1. 本教程来源于我在武汉大学的一个系列讲演, 可是教程的内容比讲演的内容扩充了很多. 我当时的主要目的是把研读群论的学生引导到能阅读和理解文章 “The hyperfocal subalgebra of a block” [10].

1.2. 这篇文章的主要结果, 即超聚焦子代数的存在性与唯一性, 在 Brauer 块理论里可能是基本的; 特别地, 块论中的另一个重要结果, 所谓幂零块的源代数的结构定理 (文章 [9] 的主要结果), 可作为它的直接推论. 后来我终于把证明简化了, 可以在一本不长的教程中完整地把它叙述出来. 这样, 读者可以看到事情的整个思想和面貌.

1.3. 差不多六十年前 Richard Brauer 引进了块理论; 他的目的是研究特征为非零素数 p 的域 k 上的有限群 G 的群代数 kG: 任意 kG 的不可分解直和项的双边理想决定一个 G-块. 可是 Brauer 的主要发现可能是: 存在无限个彼此不同构的群它们有一个 “共同块”. 当然, 我应该明确 “共同块” 这个词汇的精确含义; 事实上, 可以有不同的含义.

1.4. 我的意见: 最好方式是考虑所谓 G-块的源代数. 我自己在文章 “Pointed groups and construction of characters” [7] 里引进了源代数的定义; 原来的目的只是研究上面

1. Introduction

1.1. The present course comes from a series of lectures we gave at Wuhan University, but its contents are significantly larger. Our aim was to provide an introduction for the understanding of the paper “The hyperfocal subalgebra of a block” [10], to students in group theory.

1.2. The subject of that paper, namely the existence and the uniqueness of the *hyperfocal subalgebra of a block*, seems fundamental in Brauer block theory; for instance, an important result in block theory, *the structure of the source algebra of a nilpotent block* (the main result in [9]), can be obtained as a corollary. We subsequently succeeded in shortening the proof and decided to issue a complete and self-contained account which will allow, we hope, a better understanding of the subject.

1.3. About sixty years ago, Richard Brauer introduced the *block theory*; his purpose was to study the group algebra kG of a finite group G over a field k of nonzero characteristic p: any indecomposable two-sided ideal which is also a direct summand of kG determines a G-block. But Brauer's main discovery was perhaps the existence of families of infinitely many nonisomorphic groups having “a block in common”. Of course, the expression “a block in common” demands a definition; actually, more than one reasonable definition might exist.

1.4. Our point of view is to consider the so-called *source algebra* of a G-block, which we introduced in “Pointed groups and construction of characters” [7]; the original purpose was just to study the nilpotent blocks mentioned above, which were

提到的幂零块, 当时它已经由文章 “A Frobenius theorem for blocks” [2] 引进. 幂零块的源代数在文章 “Nilpotent blocks and their source algebras” [9] 里解决以后, 如下事实是越来越清楚了: 有关 G-块的一般结构由 G-块的源代数决定. 所以, 对我来说, 两个群有一个“共同块”的含义是它们各自有一个块其对应的源代数是相互同构的.

already introduced in “A Frobenius theorem for blocks” [2]. Once the *source algebra* of a nilpotent block was determined in “Nilpotent blocks and their source algebras” [9], it became more and more clear that the source algebra of a G-block determines everything concerning the G-block. Thus, for us, to say that two groups have “a block in common” means that we find a block in each one in such a way that the corresponding source algebras are isomorphic.

1.5. 在这本教程里, 上述技术性词汇都是细心引进的. 为了阅读它只是需要例如书 [12] 的知识, 以及 Wedderburn 定理, Nakayama 引理一类的基本代数知识; 也就是说, 在这个基础上, 证明都是完备的.

1.5. In this course, all the concepts mentioned above are carefully introduced. To follow the exposition just requires familiarity with the contents of [12], Wedderburn's Theorem, Nakayama's Lemma and other basic algebraic topics; in other words, in this text all the proofs are complete.

1.6. 为了研究 kG 与复数域上的群代数 $\mathbb{C}G$ 的关系, Brauer 考虑了在满足 $\mathcal{O}/J(\mathcal{O}) \cong k$ 的特征为零的局部环 $\mathcal{O}$ 上的群代数 $\mathcal{O}G$; 为了研究 kG 与 $\mathcal{O}G$ 的关系, 基本结果是其幂等元的关系. 所以, 对一般的 $\mathcal{O}$-代数 A, 为了从 $A/J(A)$ 到 A 提升幂等元, 在§2 里开始研究能够在 $\mathcal{O}$ 上做这件事的充分条件; 我的方法来源于 [13,II,§4].

1.6. In order to study the relationship between kG and the group algebra $\mathbb{C}G$ over the complex numbers, Brauer considers the group algebra $\mathcal{O}G$ over a local ring $\mathcal{O}$ of characteristic zero fulfilling $\mathcal{O}/J(\mathcal{O}) \cong k$; then, to relate kG and $\mathcal{O}G$, the basic result concerns the relationship between the idempotents. Thus, we start by considering in §2 sufficient conditions on $\mathcal{O}$ to lift idempotents from $A/J(A)$ to A, for any $\mathcal{O}$-algebra A; our method comes from [13,II,§4].

1.7. 一般情况下, 不需要在 k 上附加其它条件; 只有在某些论断里需要 k 是“充分大”的, 才假定 k 是代数闭的. 从§3 以后, 我假定 $\mathcal{O}$ 是完备的离散赋值环, 这就能够提升幂等元; 一般来说允许 $\mathcal{O} = k$, 可是在最后三节里 $\mathcal{O}$ 的特征应该为零.

1.7. In general, we do not need more hypotheses on k, and in those cases where we do need k be “big enough”, we simply assume that k is algebraically closed. From §3 on, we assume that $\mathcal{O}$ is a complete discrete valuation ring, which is enough for lifting idempotents; we allow the possibility $\mathcal{O} = k$, except in the last three sections where the characteristic of $\mathcal{O}$ has to be zero.

1.8. 在 §3 里，我引进所谓 $\mathcal{O}$-代数的点和 $\mathcal{O}$-代数的除子; 事实上, 对任意 $\mathcal{O}$-代数 A, 其除子和投射 A-模的同构类基本上是一样的, 可是在考虑某些 $\mathcal{O}$-代数和某些同态的, 除子的观点比较容易. 以前, 只是考虑 A 的点, 没有考虑 A 的除子; 现在从除子的观点研究, 用所谓"可嵌入同构"代替了原来的 "embeddings" (见 [7]).

1.9. 从 §4 开始考虑一个有限群 G, 和带有 G-作用的 $\mathcal{O}$-代数 A, 这种结构称为 G-代数; 事实上, 那里考虑更一般的情况. 对 G 的任意子群 H 可考虑所有 H-稳定元素构成的 $\mathcal{O}$-子代数 A^H 和它的除子, 称为 A 上的 H-除子. 那么, 重要工具是与 A 的 H-除子 ω 相关的 H-代数 A_ω. 事实上, A_ω 只是在"可嵌入同构"意义下定义的; 关于这个事实我作了细心分析.

1.10. 在 §5 可以看到考虑除子比仅仅考虑点的更有利之处; 确实, 在 A 上的 H-除子之间, 这里 H 跑遍 G 的子群, "限制"和"诱导"可以妥善处置. 关于 H-除子的诱导需要构造适当的 G-代数; 特别是, 我们得到一个 Higman 投射性条件的推广. 请注意, 文章 "Induction, restriction and G-algebras" [1] 已经引进 A 的 H-除子以及它们的限制和它们的诱导, 可是关于诱导的定义, 他的和我的不一样.

1.8. In §3, we introduce the *points* and the *divisors* on an $\mathcal{O}$-algebra A; actually, the *divisors* on A and the isomorphism classes of projective A-modules basically coincide, but when considering several $\mathcal{O}$-algebras together with several algebra homomorphisms, the *divisor* point of view is easier. Previously we only considered *points* on A; in the present course, the *divisor* point of view allows us to replace the previous "embeddings" (see [7]) by the so-called *embeddable isomorphisms*.

1.9. In §4 we already consider a finite group G, and an $\mathcal{O}$-algebra A endowed with a G-action, called a *G-algebra*; in fact, we will consider a more general situation. Thus, for any subgroup H of G, we can consider the subalgebra A^H of elements fixed by H, and the divisors of A^H, called *H-divisors* on A. Then, an important construction is the H-algebra A_ω relative to an H-divisor ω on A. Actually, A_ω is only defined up to an *embeddable isomorphism*; this is a question that we will analyse with care.

1.10. In §5, the advantage in considering *divisors* instead of merely considering *points* can already be understood; it is in the set of all the H-divisors on A, when H runs over the set of all the subgroups of G, that "restriction" and "induction" can be handled. For the induction of H-divisors, we have to construct a suitable G-algebra; in particular, we get a generalization of the Higman Projectivity Condition. The H-divisors on A, and their restrictions and inductions, have already been introduced in "Induction, restriction and G-algebras" [1], but note that the definition of the induction there does not coincide with ours.

1.11. 在§6里，我引进 A 上的所谓局部点群，以及对应于 A 上的 H-点的 H-代数 A_β 的源代数. 在 §7 里, 我证明了 Green 不可分解性定理的推广; 这个推广是已经知道的 (见 [6] 和 [14,§5]), 可是我这里的证明可能是新的. 在§8里，开始了超聚焦子代数的存在性的证明的准备工作;那里, 我在比文章 [8] 更一般的情况下引进了所谓 A-融合.

1.11. In §6, we introduce the *local pointed groups* on A, together with the *source algebra* of the H-algebra A_β associated with an H-point on A. In §7, we prove a generalization of the Green Indecomposability Theorem; this generalization is already known (see [6] and [14,§5]), but our proof here is possibly new. In §8, we begin the preparatory work for the proof of the existence of the *hyperfocal subalgebra*; here, in a more general context than in [8], we introduce the so-called *A-fusions*.

1.12. 在 A 上的 K-除子之间的 A-融合, 其中 K 跑遍 G 的子群, 是很重要的工具, 这是因为如果 A 上的某些 K-除子"包含"在 A 的 H-除子 ω 里, 那么其 A-融合在对应 H-代数 A_ω 是保存着的. 事实上, 本书 A-融合的定义比在文章 [10] 里的好多了. 由于樊恽的注意, 在§9里的结果比文章 [10] 的对应的结果更具一般性; 在§9里，对于证明超聚焦子代数的存在性来说, 我得到一个关键步骤.

1.12. The *A-fusions* between K-divisors on A, where K runs over the set of subgroups of G, is a very important tool because if some K-divisors on A are "contained" in an H-divisor ω on A then their A-fusions are preserved in the corresponding H-algebra A_ω. As a matter of fact, we give a definition of the A-fusions which improves on the formulation given in [10]. Following a remark of Fan Yun, we obtain results in §9 more general than the corresponding ones in [10]; in §9, we reach a key step in the proof for the existence of the *hyperfocal subalgebra*.

1.13. 在§10里，我把所有上述的定义和结果应用到群代数; 例如, $\mathcal{O}G$ 上的 G-点就是 G-块. 一般来说, 在群代数里上述的结果都有特种内容; 这样, 我这里再一次证明文章 [8] 里关于 $\mathcal{O}G$ 上的局部点群之间的 $\mathcal{O}G$-融合的主要结果. 事实上, $\mathcal{O}G$ 上的局部点群之间的 $\mathcal{O}G$-融合仅依赖于 $\mathcal{O}G$ 上的"少数"局部点群的 $\mathcal{O}G$-自融合, 即 $\mathcal{O}G$ 上的极大局部点群和所谓关键点群.

1.13. In §10, we apply all the definitions and results above to the case of the *group algebra;* for instance, the G-points on $\mathcal{O}G$ are just the G-blocks. In general, all the results above have a particular form in the group algebra; thus, here we prove again the main result in [8] on the $\mathcal{O}G$-fusions between the local pointed groups on $\mathcal{O}G$. Actually, the $\mathcal{O}G$-fusions between the local pointed groups on $\mathcal{O}G$ only depend on the $\mathcal{O}G$-fusions of "few" local pointed groups to themselves, like the maximal ones and the so-called *essential* local pointed groups.

1.14. 在 §11 里, 为了表示这个事实, 我引进 $\mathcal{O}G$ 上的关键点群, 以及适当的定义; $\mathcal{O}G$-融合只是依赖某些 $\mathcal{O}G$-自融合, 这个事实对于证明超聚焦子代数的存在性来说也是一个关键步骤. 在 §12 里, 我研究 G-块的源代数的特殊性; 而且, 在一个例子里我展示无限个相互不同构的群有一个"共同块"; 更确切地, 它们的所谓主块的源代数都是相互同构的.

1.14. In order to formulate this statement we introduce in §11 the *essential* pointed groups along with suitable definitions; the fact that the $\mathcal{O}G$-fusions only depend on some $\mathcal{O}G$-*autofusions* is also a key point in the proof of existence of the *hyperfocal subalgebra*. In §12, we study the particularities of the source algebra of a G-block; we exhibit an example with infinitely many pairwise nonisomorphic groups having "a block in common": the source algebras of their *principal* blocks are pairwise isomorphic.

1.15. 在 §13 里, 我在任意 G-块的源代数里正式引进超聚焦子代数; 一般来说,在 G-块的代数里 没有对应的结构: 这个事实也表现了考虑源代数的好处. 那里, 我刻画了超聚焦子代数上的局部点群, 以及它们之间的融合; 而且, 已经证明了从超聚焦子代数的存在性可推出上述的幂零块的源代数的结构定理.

1.15. In §13, we introduce the *hyperfocal subalgebra* in the source algebra of a G-block; in general, in the algebra of a G-block there is no such structure: this fact shows the advantage in considering the source algebra. There, we describe the local pointed groups on the *hyperfocal subalgebra* and the fusions between them; moreover, from the existence of the *hyperfocal subalgebra* we can already prove the theorem on the structure of the source algebras of nilpotent blocks.

1.16. 在 §14 和 §15 里, 我需要 $\mathcal{O}$-代数理论中关于 p-进分析的某些结果; 它们基本依赖于交换 $\mathcal{O}$-代数上的指数函数和对数函数的存在性. 为了不打断研究思路, 我把这些辅助的结果集中 在 §16 里.

1.16. In §14 and §15, we need some results on *p-adic* analysis; they basically depend on the existence of the *exponential* and the *logarithmic* functions in commutative $\mathcal{O}$-algebras. In order to avoid any interruption in the exposition, we collect these results in §16.

1.17. 最后, 在 §14 和 §15 里, 我证明超聚焦子代数的存在性和唯一性; 事实上, 由于用归纳法, 为了证明其存在性, 我不仅需要超聚焦子代数的唯一性, 而且需要一个更精细的结果. 在 §14 里, 我给出了这个更苛刻的唯一性; 它的证明应用了 §16 里的关于非交换上同调的一个结果: 它相当于说某个 1-上循环就是 1-上边缘.

1.17. Finally, in §14 and §15, we prove the existence and the uniqueness of the *hyperfocal subalgebra*; actually, because we argue by induction, in order to prove existence we need to frame the uniqueness in a stronger way. In §14, we prove this stronger form of uniqueness employing a result of §16 which can be considered a result on noncommutative cohomology: roughly speaking it says that some 1-*cocycle* is a 1-*coboundary*.

1.18. 在§15里, 我证明超聚焦子代数的存在性; 其关键是适当的提升的存在性 (见推论 15.9): 樊恽在 [4] 里已经得到了这种结果. 这里, 我给出的证明部分不同于在文章 [10] 里的证明; 特别是, 文章 [10] 里的证明要引用文章 [11] 的结果而这里不再需要了.

1.18. In §15, we prove the existence of the *hyperfocal subalgebra*; the key to the proof is the existence of a suitable lifting (see Corollary 15.9): in [4], Fan Yun already obtained a result of this type. The proof we give here is partially different from the proof in [10]; in particular, the proof in [10] employs some results of [11] which are no longer necessary here.

1.19. 本教程的准备和写作得到了很多人的关心与帮助: **武汉大学** 邀请. **樊恽** 教授安排了我在数学系的系列讲演; 并且他细心评阅这本教程, 既从中文的观点也从数学的观点评阅; 事实上, 是他鼓动我在本书给出超聚焦子代数的存在性和唯一性的一个完整证明. **张继平** 教授准备了一个英中数学词汇. **Joe庄** 细心评阅引言的英文的方面. **Alberto Arabia** 教授制作了计算机的特种中文软件, 让我在我的计算机里能够使用汉字; 为了得到这双栏目排版, 他还制作了另一个计算机的软件. 我的汉语老师 **茹小雷** 和我的夫人 **Isabel** 帮助我做七千多汉字的编码以及录入计算机的工作. 我真诚感谢所有这些朋友和亲人.

1.19. The elaboration of this course has been made possible thanks to the collaboration of many people: The invitation by **Wuhan University**. The organization of the series of lectures by **Fan Yun**, who revised both the Chinese writing and the mathematical content in this course with great care, and encouraged me to issue a complete account on the *hyperfocal subalgebra*. **Zhang Ji Ping** prepared for me a mathematical English-Chinese vocabulary. **Joe Chuang** carefully revised the English part of this introduction. **Alberto Arabia** created a computer program allowing me to employ Chinese characters in my usual TeX program, and a second one for the two-column pattern. My Chinese language teacher, **Ru Xiao Lei**, and my wife **Isabel** helped me to implement the codes of about seven thousand Chinese characters on the computer. I deeply thank them all.

2. 幂等元的提升

2.1. 为了从特征为素数 p 提升一个幂等元到特征为零, 有效的方法是使用一个特征为零的完备的离散赋值环使得其根上的商环是特征为 p 的域. 事实上, 完备化条件是允分的但不必要的. 下面, 我考虑这些问题.

2.2. 设 $\mathcal{O}$ 是一个离散赋值环使得 $k = \mathcal{O}/J(\mathcal{O})$ 是特征为 p 的域, 其中 $J(\mathcal{O})$ 是 $\mathcal{O}$ 的根基, 并且 $\mathcal{O}$ 的分式域 $\mathcal{K}$ 是为特征零的域. 也就是说, 有一个满射的群同态 $\vartheta: \mathcal{K}^* \to \mathbb{Z}$ 使得

特别是, 有 $\pi \in \mathcal{O}$ 使得 $J(\mathcal{O}) = \pi\mathcal{O}$. 请注意, 如果 $\mathcal{K}'$ 是 $\mathcal{K}$ 的 Galois 扩张, 那么从范数映射 $\mathfrak{N}: \mathcal{K}'^* \to \mathcal{K}^*$ 不难定义一个满射的群同态 $\vartheta': \mathcal{K}'^* \to \mathbb{Z}$; 进一步, 不难证明下面的集合

就是 $\mathcal{K}'$ 中 $\mathcal{O}$ 上的整元构成的环 $\mathcal{O}'$; 也就是说, $\mathcal{O}'$ 也是离散赋值环.

2.3. 设 $\{\lambda_n\}_{n\in\mathbb{N}}$ 是一个 $\mathcal{K}$ 的元素的序列; 如果存在 $\lambda \in \mathcal{K}$ 使得对任意 $n \in \mathbb{N}$ 有 $\lambda - \lambda_n \in \pi^{n+1}\mathcal{O}$, 那么我们说 $\{\lambda_n\}_{n\in\mathbb{N}}$ 有极限 λ, 并且我们记 $\lambda = \lim_{n\to\infty}\{\lambda_n\}$ (请注意, 这个条件比通常的收敛条件更强). 例子: 如果对任意 $n \in \mathbb{N}$ 有 $\lambda_n = \lambda$, 这个序列当然有极限 λ. 有极限的序列满足所谓 Cauchy 条件;

2. Lifting Idempotents

2.1. In order to lift idempotents from characteristic p to characteristic zero, the safest method is to work over a complete discrete valuation ring of characteristic zero with a residue field of characteristic p. Yet, as a matter of fact, the completeness is a sufficient but not necessary condition. In this section, we will discuss on this question.

2.2. Let $\mathcal{O}$ be a discrete valuation ring such that $k = \mathcal{O}/J(\mathcal{O})$ is a field of characteristic p, where $J(\mathcal{O})$ denotes the radical of $\mathcal{O}$, and that its field of quotients $\mathcal{K}$ has characteristic zero. That is to say, we have a surjective group homomorphism $\vartheta: \mathcal{K}^* \to \mathbb{Z}$ fulfilling

2.2.1 $$\{\lambda \in \mathcal{K}^* \mid \vartheta(\lambda) \geq 0\} = \mathcal{O} - \{0\};$$

in particular, there is $\pi \in \mathcal{O}$ such that $J(\mathcal{O}) = \pi\mathcal{O}$. Note that, if $\mathcal{K}'$ is a Galois extension of $\mathcal{K}$, then from the norm map $\mathfrak{N}: \mathcal{K}'^* \to \mathcal{K}^*$ it is not difficult to define a surjective group homomorphism $\vartheta': \mathcal{K}'^* \to \mathbb{Z}$; moreover, it is easy to check that the set

2.2.2 $$\{\lambda' \in \mathcal{K}'^* \mid \mathfrak{N}(\lambda') \in \mathcal{O}\} \cup \{0\}$$

coincides with the integral closure $\mathcal{O}'$ of $\mathcal{O}$ in $\mathcal{K}'$; in other words, $\mathcal{O}'$ still is a discrete valuation ring.

2.3. Let $\{\lambda_n\}_{n\in\mathbb{N}}$ be a sequence of elements of $\mathcal{K}$; whenever there is $\lambda \in \mathcal{K}$ such that we have $\lambda - \lambda_n \in \pi^{n+1}\mathcal{O}$ for any $n \in \mathbb{N}$, we say that the sequence $\{\lambda_n\}_{n\in\mathbb{N}}$ has the *limit* λ, and we write $\lambda = \lim_{n\to\infty}\{\lambda_n\}$ (note that this condition is stronger than the usual one). For instance, if for any $n \in \mathbb{N}$ we have $\lambda_n = \lambda$, this sequence obviously has the limit λ. The sequences having a limit fulfill the so-called Cauchy condition; in particular, for any

特别是, 对任意 $n \in \mathbb{N}$ 有

$n \in \mathbb{N}$, we have

2.3.1 $$\lambda_{n+1} - \lambda_n \in \pi^{n+1}\mathcal{O}$$

(事实上, 上面的条件比 Cauchy 条件更强). 反过来, 如果满足这个条件的 $\mathcal{K}$-序列都有极限, 那么我们说 $\mathcal{K}$ 和 $\mathcal{O}$ 是完备的.

(actually, this condition is stronger than Cauchy's condition). Conversely, if all the $\mathcal{K}$-sequences fulfilling this condition have a limit then we say that $\mathcal{K}$ and $\mathcal{O}$ are *complete*.

2.4. 不难造一个包含 $\mathcal{K}$ 的完备域 $\hat{\mathcal{K}}$. 我们在所有满足条件 2.3.1 的 $\mathcal{K}$-序列里考虑下面的等价关系: 如果该集合里的两个序列 $\{\lambda_n\}_{n\in\mathbb{N}}$ 和 $\{\mu_n\}_{n\in\mathbb{N}}$ 满足

2.4. It is not difficult to construct a complete field $\hat{\mathcal{K}}$ containing $\mathcal{K}$. We consider the following equivalence relationship in the set of all the sequences fulfilling condition 2.3.1: if two sequences $\{\lambda_n\}_{n\in\mathbb{N}}$ and $\{\mu_n\}_{n\in\mathbb{N}}$ of this set fulfill

2.4.1 $$\mu_n - \lambda_n \in \pi^{n+1}\mathcal{O}$$

就说这两个序列是等价的. 例子: 固定 $\ell \in \mathbb{N}$, 如果 $\mu_n = \lambda_\ell$ 或 λ_n 当 $n \le \ell$ 或 $n \ge \ell$, 那么这两个序列是等价的; 又, $\{\lambda_{\ell+n}\}_{n\in\mathbb{N}}$ 也满足条件 2.3.1 并 $\{\lambda_n\}_{n\in\mathbb{N}}$ 和它是等价的. 请注意, 如果 $\{\lambda_n\}_{n\in\mathbb{N}}$ 没有极限 0, 那么对适当的 ℓ, λ_ℓ 不属于 $\pi^{\ell+1}\mathcal{O}$; 此时, 从条件 2.3.1 可推出 $\vartheta(\lambda_{\ell+n}) = \vartheta(\lambda_\ell)$, 其中 $n \in \mathbb{N}$ (cf. 2.2.1).

we say that they are equivalent. For instance, fixing $\ell \in \mathbb{N}$, if $\mu_n = \lambda_\ell$ or λ_n according to $n \le \ell$ or $n \ge \ell$ then these sequences are equivalent; moreover, $\{\lambda_{\ell+n}\}_{n\in\mathbb{N}}$ also fulfills condition 2.3.1 and clearly is equivalent to $\{\lambda_n\}_{n\in\mathbb{N}}$. Note that, if $\{\lambda_n\}_{n\in\mathbb{N}}$ has not the limit 0 then, for a suitable ℓ, λ_ℓ does not belong to $\pi^{\ell+1}\mathcal{O}$; thus, from condition 2.3.1 we get $\vartheta(\lambda_{\ell+n}) = \vartheta(\lambda_\ell)$ for any $n \in \mathbb{N}$ (cf. 2.2.1).

2.5. 设 $\hat{\mathcal{K}}$ 是所有满足条件 2.3.1 的 $\mathcal{K}$-序列的等价类的集合; 显然, 序列的加法在 $\hat{\mathcal{K}}$ 里决定一个交换群结构. 而且, 设 $\hat{\lambda} = \widetilde{\{\lambda_n\}}_{n\in\mathbb{N}}$ 与 $\hat{\mu} = \widetilde{\{\mu_n\}}_{n\in\mathbb{N}}$ 是 $\hat{\mathcal{K}}$ 的两个非等于零元素; 我们能假定对任意 $n \in \mathbb{N}$ 有 $\vartheta(\lambda_n) = \vartheta(\lambda_\circ)$ 与 $\vartheta(\mu_n) = \vartheta(\mu_\circ)$; 取 $\ell \in \mathbb{N}$ 满足 $-\vartheta(\lambda_\circ) \le \ell$ 与 $-\vartheta(\mu_\circ) \le \ell$; 此时, 因为下面的差

2.5. Let $\hat{\mathcal{K}}$ be the set of equivalent classes of sequences of $\mathcal{K}$ which fulfill condition 2.3.1; clearly, the usual sum of sequences determines a structure of commutative group in $\hat{\mathcal{K}}$. Moreover, let $\hat{\lambda} = \widetilde{\{\lambda_n\}}_{n\in\mathbb{N}}$ and $\hat{\mu} = \widetilde{\{\mu_n\}}_{n\in\mathbb{N}}$ be two nonzero elements of $\hat{\mathcal{K}}$; by the remarks above, we may assume that, for any $n \in \mathbb{N}$, we have $\vartheta(\lambda_n) = \vartheta(\lambda_\circ)$ and $\vartheta(\mu_n) = \vartheta(\mu_\circ)$; choose ℓ fulfilling $-\vartheta(\lambda_\circ) \le \ell$ and $-\vartheta(\mu_\circ) \le \ell$; at that point, since the difference

2.5.1 $$\lambda_{n+1}\mu_{n+1} - \lambda_n\mu_n = \lambda_{n+1}(\mu_{n+1} - \mu_n) + (\lambda_{n+1} - \lambda_n)\mu_n\,,$$

显然属于 $\pi^{n+1-\ell}\mathcal{O}$, 所以序列 $\{\lambda_{\ell+n}\,\mu_{\ell+n}\}_{n\in\mathbb{N}}$ 满足2.3.1; 那么, 不难验证 $\{\lambda_{\ell+n}\,\mu_{\ell+n}\}_{n\in\mathbb{N}}$ 的等价类,记 $\hat{\lambda}\hat{\mu}$,不依赖我们的选择.

belongs to $\pi^{n+1-\ell}\mathcal{O}$, $\{\lambda_{\ell+n}\,\mu_{\ell+n}\}_{n\in\mathbb{N}}$ fufills 2.3.1; then, it is not difficut to check that the equivalent class of $\{\lambda_{\ell+n}\,\mu_{\ell+n}\}_{n\in\mathbb{N}}$, noted $\hat{\lambda}\hat{\mu}$, does not depend on our choice.

2.6. 这个乘法运算在 $\hat{\mathcal{K}}-\{0\}$ 里决定一个交换群结构; 确实, 不难验证结合律. 另一方面, 令 v 记 $\vartheta(\lambda_\circ)$ 与 0 中较大的数; 不难验证有

2.6. This operation determines a commutative group structure in $\hat{\mathcal{K}}-\{0\}$; indeed, the associativity is easily checked. On the other hand, denote by v the biggest of $\vartheta(\lambda_\circ)$ and 0; we clearly have

2.6.1 $$\lambda_{v+n} = \lambda_v(1+\sum_{i=1}^{n}\delta_i\pi^i)$$

其中 $n\in\mathbb{N}$ 与 $\delta_i\in\mathcal{O}$; 用归纳法我们能定义下面的序列 $\{\mu_n\}_{n\in\mathbb{N}}$

where $n\in\mathbb{N}$ and $\delta_i\in\mathcal{O}$; arguing by induction, we can define the following sequence $\{\mu_n\}_{n\in\mathbb{N}}$

2.6.2 $$\mu_n = 1+\sum_{i=1}^{n}\varepsilon_i\pi^i\,, \quad \varepsilon_i = -\delta_i-\sum_{j=1}^{i-1}\delta_j\varepsilon_{i-j}\,;$$

因为对任意 $i\in\mathbb{N}$ 有 $\varepsilon_i\in\mathcal{O}$, 所以它满足条件 2.3.1; 此时, 不难验证其等价类乘以 λ_v^{-1} 常数序列的等价类就是 $\hat{\lambda}$ 的逆元素.

since we have $\varepsilon_i\in\mathcal{O}$ for any $i\in\mathbb{N}$, this sequence fulfills condition 2.3.1; thus, it is easy to check that its equivalent class multiplied by the equivalent class of the constant sequence λ_v^{-1} is the inverse of $\hat{\lambda}$.

2.7. 当然, 我定义 $0\hat{\lambda}=0=\hat{\lambda}0$; 那么不难验证分配律. 这样, 就证明了 $\hat{\mathcal{K}}$ 是一个域, 而只要把 $\lambda\in\mathcal{K}$ 与 λ 常数序列的等价类等同一致, $\mathcal{K}$ 就是一个 $\hat{\mathcal{K}}$ 的子域. 在 $\hat{\mathcal{K}}$ 里, 设 $\hat{\mathcal{O}}$ 是所有 $\mathcal{O}$-序列的等价类的集合; 显然, $\hat{\mathcal{O}}$ 是 $\hat{\mathcal{K}}$ 的子环, 并且它包含 $\mathcal{O}$.

2.7. Obviously, we set $0\hat{\lambda}=0=\hat{\lambda}0$; now, it is not difficult to check the distributivity. Hence, we have proved that $\hat{\mathcal{K}}$ is a field, and it suffices to identify $\lambda\in\mathcal{K}$ with the equivalent class of the constant sequence λ to get $\mathcal{K}$ as a subfield of $\hat{\mathcal{K}}$. Let $\hat{\mathcal{O}}$ be the subset of $\hat{\mathcal{K}}$ of all the equivalent classes of sequences in $\mathcal{O}$; clearly, $\hat{\mathcal{O}}$ is a subring of $\hat{\mathcal{K}}$ which contains $\mathcal{O}$.

2.8. 而且, 我们能把 $\mathcal{K}$ 的离散赋值 $\vartheta\colon\mathcal{K}^*\to\mathbb{Z}$ 扩张为一个 $\hat{\mathcal{K}}$ 的离散赋值 $\hat{\vartheta}$ 如下: 设 $\hat{\lambda}=\widetilde{\{\lambda_n\}}_{n\in\mathbb{N}}$ 是一个 $\hat{\mathcal{K}}^*$ 的元素; 别忘了, 我们能假定对任

2.8. Moreover, we can extend the discrete valuation $\vartheta\colon\mathcal{K}^*\to\mathbb{Z}$ of $\mathcal{K}$ to a discrete valuation $\hat{\vartheta}$ of $\hat{\mathcal{K}}$ as follows: let $\hat{\lambda}=\widetilde{\{\lambda_n\}}_{n\in\mathbb{N}}$ be an element of $\hat{\mathcal{K}}^*$; recall that we may assume that we have $\vartheta(\lambda_n)=\vartheta(\lambda_\circ)$ for any

意 $n \in \mathbb{N}$ 有 $\vartheta(\lambda_n) = \vartheta(\lambda_\circ)$；那么就令 $\hat{\theta}(\hat{\lambda}) = \theta(\lambda_\circ)$；特别是，$\hat{\theta}(\hat{\lambda}) \geq 0$ 当且仅当 $\hat{\lambda} \in \hat{\mathcal{O}}$.

$n \in \mathbb{N}$; in this case, we just set $\hat{\theta}(\hat{\lambda}) = \theta(\lambda_\circ)$; in particular, note that we have $\hat{\theta}(\hat{\lambda}) \geq 0$ if and only if $\hat{\lambda} \in \hat{\mathcal{O}}$.

2.9. 最后，我断言 $\hat{\mathcal{K}}$ 和 $\hat{\mathcal{O}}$ 是完备的. 设 $\{\hat{\lambda}_n\}_{n\in\mathbb{N}}$ 是 $\hat{\mathcal{K}}$ 的一个元素的序列使得对任意 $n \in \mathbb{N}$ 有

2.9. Finally, we claim that $\hat{\mathcal{K}}$ and $\hat{\mathcal{O}}$ are complete. Let $\{\hat{\lambda}_n\}_{n\in\mathbb{N}}$ be a sequence of elements of $\hat{\mathcal{K}}$ such that, for any $n \in \mathbb{N}$, we have

2.9.1 $$\hat{\lambda}_{n+1} - \hat{\lambda}_n \in \pi^{n+1}\hat{\mathcal{O}};$$

这样，如果 $\hat{\lambda}_n = \widetilde{\{\lambda_{n,m}\}}_{m\in\mathbb{N}}$，对适当的 $\mu_{n,m} \in \mathcal{O}$，其中 $n,m \in \mathbb{N}$，还有

thus, if $\hat{\lambda}_n = \widetilde{\{\lambda_{n,m}\}}_{m\in\mathbb{N}}$ then, for a suitable choice of $\mu_{n,m} \in \mathcal{O}$, where $n,m \in \mathbb{N}$, we still have

2.9.2 $$(\lambda_{n+1,m} - \lambda_{n,m}) - \pi^{n+1}\mu_{n,m} \in \pi^{m+1}\mathcal{O};$$

别忘了，由 2.4 的子里，能假定对任意 $m \leq n$ 有 $\lambda_{n,m} = \lambda_{n,n+1}$；此时，元素 $\lambda_{n+1,m} - \lambda_{n,m}$ 属于 $\pi^{n+1}\mathcal{O}$，其中 $m,n \in \mathbb{N}$；因为这些序列满足条件 2.3.1，所以对任意 $n \in \mathbb{N}$ 下面的差

recall that, according to the example in 2.4, we may assume that $\lambda_{n,m} = \lambda_{n,n+1}$ for any $m \leq n$; then, for any $m,n \in \mathbb{N}$, we obtain $\lambda_{n+1,m} - \lambda_{n,m} \in \pi^{n+1}\mathcal{O}$; since all these sequences fulfill condition 2.3.1, for any $n \in \mathbb{N}$, the difference

2.9.3 $$\lambda_{n+1,n+1} - \lambda_{n,n} = \lambda_{n+1,n+1} - \lambda_{n+1,n} + \lambda_{n+1,n} - \lambda_{n,n};$$

属于 $\pi^{n+1}\mathcal{O}$；这样，$\{\lambda_{n,n}\}_{n\in\mathbb{N}}$ 满足 2.3.1，从而这个序列决定 $\hat{\mathcal{K}}$ 的一个元素 $\hat{\lambda}$.

belongs to $\pi^{n+1}\mathcal{O}$; thus, $\{\lambda_{n,n}\}_{n\in\mathbb{N}}$ fulfills condition 2.3.1 and therefore determines an element $\hat{\lambda}$ of $\hat{\mathcal{K}}$.

2.10. 进一步，对任意 $m,n \in \mathbb{N}$ 只要 $m \geq n$ 就有

2.10. Moreover, for any $m,n \in \mathbb{N}$, it suffices that $m \geq n$ to get

2.10.1 $$\lambda_{m,m} - \lambda_{n,m} = \sum_{i=n}^{m-1} (\lambda_{i+1,m} - \lambda_{i,m}) \in \pi^{n+1}\mathcal{O};$$

所以，只要把上面的例子应用到序列 $\{\lambda_{m,m}\}_{m\in\mathbb{N}}$，就 $\hat{\lambda} - \hat{\lambda}_n$ 属于 $\pi^{n+1}\hat{\mathcal{O}}$，其中 $n \in \mathbb{N}$；也就是说，序列 $\{\hat{\lambda}_n\}_{n\in\mathbb{N}}$ 有极限 $\hat{\lambda}$. 请注意，如果 $\{\lambda_n\}_{n\in\mathbb{N}}$ 在 $\mathcal{K}$ 中是满足条件 2.3.1 的序列，那么 $\{\lambda_n\}_{n\in\mathbb{N}}$ 的等价类在 $\hat{\mathcal{K}}$ 中就是这个序列的极限；所以只要 $\mathcal{K}$ 是完备的，就得到 $\hat{\mathcal{K}} = \mathcal{K}$.

hence, it suffices to apply the example above to the sequence $\{\lambda_{m,m}\}_{m\in\mathbb{N}}$ to get that $\hat{\lambda} - \hat{\lambda}_n$ belongs to $\pi^{n+1}\hat{\mathcal{O}}$ for any $n \in \mathbb{N}$; that is, the sequence $\{\hat{\lambda}_n\}_{n\in\mathbb{N}}$ has the limit $\hat{\lambda}$. Note that, if $\{\lambda_n\}_{n\in\mathbb{N}}$ is a sequence in $\mathcal{K}$ which fulfills condition 2.3.1, then the element of $\hat{\mathcal{K}}$ determined by the equivalent class of $\{\lambda_n\}_{n\in\mathbb{N}}$ actually is the limit of this sequence; hence, if $\mathcal{K}$ is complete then we simply get $\hat{\mathcal{K}} = \mathcal{K}$.

2.11. **我们总是假定 $\mathcal{O}$-代数都是有限秩的自由 $\mathcal{O}$-模.** 设 A 是一个 $\mathcal{O}$-代数; 令 J 记 A 的根. 已经知道, 如果 $\mathcal{O}$ 是完备的, 那么 A-序列 $\{a_n\}_{n\in\mathbb{N}}$ 只要满足 $a_{n+1}-a_n\in J^{n+1}$, 其中 $n\in\mathbb{N}$, 它就有极限; 也就是说, 存在 $a\in A$ 使得对任意 $n\in\mathbb{N}$ 有 $a-a_n\in J^{n+1}$. 确实, 对适当的 $r\in\mathbb{N}$ 有 $J^r\subset\pi.A$, 从而对任意 $n\in\mathbb{N}$ 下面的差

2.11. *We always assume that the $\mathcal{O}$-algebras are $\mathcal{O}$-free $\mathcal{O}$-modules of finite rank.* Let A be an $\mathcal{O}$-algebra; denote by J the radical of A. It is well-known that if $\mathcal{O}$ is complete then it suffices that a sequence $\{a_n\}_{n\in\mathbb{N}}$ in A fulfills $a_{n+1}-a_n\in J^{n+1}$ for any $n\in\mathbb{N}$, to guarantee that it has a limit; precisely, in that case there is $a\in A$ such that $a-a_n\in J^{n+1}$ for any $n\in\mathbb{N}$. Indeed, we have $J^r\subset\pi.A$ for a suitable $r\in\mathbb{N}$ and therefore, for any $n\in\mathbb{N}$, the difference

2.11.1 $$a_{(r+1)(n+1)}-a_{(r+1)n}=\sum_{i=0}^{r}(a_{(r+1)n+i+1}-a_{(r+1)n+i})$$

属于 $J^{(r+1)n+1}$; 因为 A 是有限秩的自由 $\mathcal{O}$-模并对任意 $n\geq r$ 有 $J^{(r+1)n+1}\subset\pi^{n+1}\cdot A$, 所以存在 $a\in A$ 使得对任意 $n\geq r$ 有

belongs to $J^{(r+1)n+1}$; since A is an $\mathcal{O}$-free $\mathcal{O}$-module of finite rank and, for any $n\geq r$, we have $J^{(r+1)n+1}\subset\pi^{n+1}\cdot A$, there is $a\in A$ such that, for any $n\geq r$, we have

2.11.2 $$a-a_{(r+1)n}\in\pi^{n+1}.A\subset J^{n+1},$$

从而还有

and therefore we also have

2.11.3 $$a-a_n=a-a_{(r+1)n}+\sum_{i=n+1}^{(r+1)n}(a_i-a_{i-1})\in J^{n+1};$$

进一步, 对任意 $n\leq r$ 也得到

moreover, for any $n\in\mathbb{N}$, we still obtain

2.11.4 $$a-a_n=a-a_r+\sum_{i=n}^{r-1}(a_{i+1}-a_i)\in J^{n+1}.$$

2.12. 特别是, 如果 A' 是 $\mathcal{O}$-代数并 f 是 $\mathcal{O}$-代数同态从 A 到 A' 使得对任意 $n\in\mathbb{N}$ 有 $f(a_{n+1})-f(a_n)\in J'^{n+1}$ 其中 J' 是 A' 的根基,那么 $f(a)-f(a_{rn})$ 属于 $\pi^{n+1}.A'\subset J'^{n+1}$, 从而得到 $f(a)-f(a_n)\in J'^{n+1}$; 也就是说, $\{f(a_n)\}_{n\in\mathbb{N}}$ 有极限 $f(a)$. 而且如果 $r\in J$ 那么 $\{\sum_{\ell=0}^n r^\ell\}_{n\in\mathbb{N}}$ 有极限, 记为 $\sum_{\ell\in\mathbb{N}}r^\ell$; 这个极限就是 $1-r$ 的逆元素; 从而, A^* 包含 $1+J$.

2.12. In particular, if A' is an $\mathcal{O}$-algebra and f an $\mathcal{O}$-algebra homomorphism from A to A' such that, for any $n\in\mathbb{N}$, we have $f(a_{n+1})-f(a_n)\in J'^{n+1}$, where J' is the radical of A', then $f(a)-f(a_{rn})$ belongs to $\pi^{n+1}.A'\subset J'^{n+1}$ and therefore we get $f(a)-f(a_n)\in J'^{n+1}$; in other terms, $\{f(a_n)\}_{n\in\mathbb{N}}$ has the limit $f(a)$. Moreover, if $r\in J$ then the sequence $\{\sum_{\ell=0}^n r^\ell\}_{n\in\mathbb{N}}$ has a limit, noted $\sum_{\ell\in\mathbb{N}}r^\ell$; this limit actually coincides with the inverse of $1-r$; consequently, A^* contains $1+J$. Note that,

请注意, 如果 $\mathcal{K}'$ 是 $\mathcal{K}$ 的 Galois 扩张, 并 $\mathcal{O}'$ 是 $\mathcal{K}'$ 中 $\mathcal{O}$ 上的整元的环 $\mathcal{O}'$, 那么 $\mathcal{O}'$ 与 $\mathcal{K}'$ 也是完备的.

if $\mathcal{K}'$ is a Galois extension of $\mathcal{K}$, and $\mathcal{O}'$ is the integral closure of $\mathcal{O}$ in $\mathcal{K}'$, then $\mathcal{O}'$ and $\mathcal{K}'$ are complete too.

定理 2.13. 假定 $\mathcal{O}$ 是完备的. 设 A 是一个交换 $\mathcal{O}$-代数并设 I 是 A 的理想. 令 J 记 A 的根基并 $s: A \to A/J$ 记自然的映射, 再令

Theorem 2.13. *Assume that $\mathcal{O}$ is complete. Let A be a commutative $\mathcal{O}$-algebra and I an ideal of A. Denote by J the radical of A and by $s: A \to A/J$ the canonical map, and set* †

2.13.1
$$\text{幂}^{p^{\aleph}}(I) = \bigcap_{n\in\mathbb{N}} \left(\{a^{p^n} \mid a \in I\} + J^{n+1}\right),$$
$$\text{幂}^{p^{\aleph}}\big(s(I)\big) = \bigcap_{n\in\mathbb{N}} \{s(a)^{p^n} \mid a \in I\}.$$

那么 s 决定一个从 $\text{幂}^{p^{\aleph}}(I)$ 到 $\text{幂}^{p^{\aleph}}\big(s(I)\big)$ 的双射; 而且, 有

Then, s determines a bijection between $\text{幂}^{p^{\aleph}}(I)$ and $\text{幂}^{p^{\aleph}}\big(s(I)\big)$; moreover, we have

2.13.2
$$\text{幂}^{p^{\aleph}}(I) \cdot \text{幂}^{p^{\aleph}}(I) \subset \text{幂}^{p^{\aleph}}(I).$$

证明: 请注意, 如果 $a, b \in A$ 满足 $s(a) = s(b)$, 那么对任意 $n \in \mathbb{N}$ 有 $a^{p^n} - b^{p^n} \in J^{n+1}$; 确实, 只要使用归纳法, 我们就能假定 $n > 1$ 并且 $c = a^{p^{n-1}} - b^{p^{n-1}}$ 属于 J^n; 显然有

Proof: First of all, note that if $a, b \in A$ fulfill the equality $s(a) = s(b)$ then, for any $n \in \mathbb{N}$, we have $a^{p^n} - b^{p^n} \in J^{n+1}$; indeed, we argue by induction on n and may assume that $n > 1$ and that the element $c = a^{p^{n-1}} - b^{p^{n-1}}$ belongs to J^n; clearly, we have

2.13.3
$$a^{p^n} = (b^{p^{n-1}} + c)^p \in b^{p^n} + pJ^n + J^{np} \subset b^{p^n} + J^{n+1}.$$

现在, 如果 $a, b \in \text{幂}^{p^{\aleph}}(I)$ 那么对任意 $n \in \mathbb{N}$ 存在 $a_n, b_n \in I$ 与 $r_n, s_n \in J^{n+1}$ 使得

Now, if $a, b \in \text{幂}^{p^{\aleph}}(I)$ then for any $n \in \mathbb{N}$, we can find $a_n, b_n \in I$ and $r_n, s_n \in J^{n+1}$ such that

2.13.4
$$a = (a_n)^{p^n} + r_n, \quad b = (b_n)^{p^n} + s_n;$$

特别是, ab 属于下面的交

in particular, ab belongs to the intersection

2.13.5
$$\bigcap_{n\in\mathbb{N}} \left((a_n b_n)^{p^n} + J^{n+1}\right);$$

从而 ab 也属于 $\text{幂}^{p^{\aleph}}(I)$. 而且, 如果有 $s(a) = s(b)$ 那么还有

hence, ab still belongs to $\text{幂}^{p^{\aleph}}(I)$. Moreover, if we have $s(a) = s(b)$ then $s(a_n)^{p^n} = s(b_n)^{p^n}$

† The Chinese character 幂 is pronounced "mi" as in "middle" and means "power" or "exponent".

$s(a_n)^{p^n} = s(b_n)^{p^n}$，从而 $0 = s(a_n - b_n)^{p^n}$，这是因为 A/J 是特征 p 的. 另一方面, 因为 A/J 是域的直积所以 $s(a_n) = s(b_n)$; 这样, 得到

and therefore we still have $s(a_n - b_n)^{p^n} = 0$ since A/J has characteristic p. On the other hand, since A/J is a direct product of fields, we also get $s(a_n) = s(b_n)$; thus, we obtain

2.13.6 $$a - b \in \bigcap_{m\in\mathbb{N}} J^m \subset \bigcap_{m\in\mathbb{N}} \pi^m \cdot A = \{0\}.$$

所以 s 决定一个单映射从 幂$^{p^{\mathbb{N}}}(I)$ 到 幂$^{p^{\mathbb{N}}}(s(I))$; 我们要证明这个映射也是满射. 设 $\bar{a}$ 是一个 幂$^{p^{\mathbb{N}}}(s(I))$ 的元素; 也就是说, 对任意 $n\in\mathbb{N}$ 存在 $a_n\in I$ 使得 $s(a_n)^{p^n} = \bar{a}$; 特别是, 有

Hence, s determines an injective map from 幂$^{p^{\mathbb{N}}}(I)$ to 幂$^{p^{\mathbb{N}}}(s(I))$; we will prove that this map is surjective too. Let $\bar{a}$ be an element of 幂$^{p^{\mathbb{N}}}(s(I))$; explicitly, this means that, for any $n \in \mathbb{N}$, there is $a_n \in I$ such that $s(a_n)^{p^n} = \bar{a}$; in particular, we get

2.13.7 $$s\big((a_{n+1})^p\big)^{p^n} = \bar{a} = s(a_n)^{p^n}$$

仍然因为 A/J 就是特征为 p 的域的直积, 所以我们得到 $s\big((a_{n+1})^p\big) = s(a_n)$; 从而, 也得到

and therefore, since A/J is a direct product of fields of characteristic p, we have $s\big((a_{n+1})^p\big) = s(a_n)$; consequently, we also obtain

2.13.8 $$(a_{n+1})^{p^{n+1}} - (a_n)^{p^n} = \big((a_{n+1})^p\big)^{p^n} - (a_n)^{p^n} \in J^{n+1}.$$

那么存在 $a \in I$ 使得对任意 $n > 0$ 有 $a - (a_n)^{p^n} \in J^{n+1}$ (见 2.11), 从而得到

Then, there is $a \in I$ such that, for any $n \in \mathbb{N}$, we have $a - (a_n)^{p^n} \in J^{n+1}$ (see 2.11), and therefore we get

2.13.9 $$s(a) = \big(s(a_n)\big)^{p^n} = \bar{a};$$

也就是说, $s(a)$ 属于 幂$^{p^{\mathbb{N}}}(s(I))$.

that is to say, $s(a)$ belongs to 幂$^{p^{\mathbb{N}}}(s(I))$.

推论 2.14. **假定 $\mathcal{O}$ 是完备的. 设 A 是 $\mathcal{O}$-代数并设 I 是一个 A 的理想. 令 J 记 A 的根, 再令 $s: A \to \bar{A}$ 为自然的映射, 其中 $\bar{A} = A/J$. 对任意幂等元 $\bar{\imath} \in \bar{I}$, 其中 $\bar{I} = s(I)$, 存在一个幂等元 $i \in A$ 使得 $s(i) = \bar{\imath}$. 而且, 如果 $i' \in A$ 是一个幂等元使得 $s(i') = \bar{\imath}$ 有 $a \in A^*$ 使得 $i' = i^a$.**

Corollary 2.14. *Assume that $\mathcal{O}$ is complete. Let A be an $\mathcal{O}$-algebra and I an ideal of A. Denote by J the radical of A and by $s: A \longrightarrow \bar{A}$ the canonical map, where $\bar{A} = A/J$. For any idempotent $\bar{\imath} \in \bar{I}$, where $\bar{I} = s(I)$, there exists an idempotent $i \in A$ such that $s(i) = \bar{\imath}$. Moreover, if $i' \in A$ is an idempotent such that $s(i') = \bar{\imath}$, then there exists $a \in A^*$ such that $i' = i^a$.*

证明: 显然有 $a \in A$ 使得 $s(a) = \bar{\imath}$; 令 $B = \sum_{n\in\mathbb{N}} \mathcal{O}\cdot a^n$;

Proof: Clearly there is $a \in A$ such that $s(a) = \bar{\imath}$; set $B = \sum_{n\in\mathbb{N}} \mathcal{O}\cdot a^n$; then B is a

那么 B 是 A 的子代数, 而有 $B \cap J = J(B)$; 所以, 只要用 $B, B\cap I$ 分别代替 A, I 就能假定 A 是交换的. 那么, 由定理 2.13, 既然 $\bar{\imath}$ 属于幂$^{p^{\mathbb{N}}}(s(I))$ 所以存在 $i \in$ 幂$^{p^{\mathbb{N}}}(I)$ 满足 $s(i) = \bar{\imath}$, 还满足 $s(i^2) = \bar{\imath}^2 = \bar{\imath}$; 仍使用定理 2.13, 既然 i^2 也属于幂$^{p^{\mathbb{N}}}(I)$, 所以 $i^2 = i$. 最后, 如果 $i' \in A$ 是一个幂等元使得 $s(i') = \bar{\imath}$, 那么令 $a = ii' + (1-i)(1-i')$; 一方面, 显然 $ia = ai'$, 另一方面有

subalgebra of A and we have $B \cap J = J(B)$; consequently, up to the replacement of A and I by B and $B \cap I$, we may assume that A is commutative. Then, according to Theorem 2.13, since $\bar{\imath}$ belongs to 幂$^{p^{\mathbb{N}}}(s(I))$, there exists $i \in$ 幂$^{p^{\mathbb{N}}}(I)$ fulfilling $s(i) = \bar{\imath}$, and in particular $s(i^2) = \bar{\imath}^2 = \bar{\imath}$; according to Theorem 2.13 again, since i^2 also belongs to 幂$^{p^{\mathbb{N}}}(I)$, we get $i^2 = i$. Finally, if $i' \in A$ is an idempotent fulfilling $s(i') = \bar{\imath}$, then consider the element $a = ii' + (1-i)(1-i')$; on the one hand, we clearly have $ia = ai'$; on the other hand, we get

2.14.1 $$s(a) = \bar{\imath}^2 + (1-\bar{\imath})^2 = 1\,,$$

从而, 还有 $a \in 1 + J \subset A^*$ (见 2.11).

and therefore we still get $a \in 1 + J \subset A^*$ (see 2.11).

2.15. 一般的说, 因为一个 $\mathcal{O}$-代数 A 是自由 $\mathcal{O}$-模, 我们能把 $a \in A$ 与 $1 \otimes a \in \hat{\mathcal{O}} \otimes_{\mathcal{O}} A$ 或者 $1 \otimes a \in \mathcal{K} \otimes_{\mathcal{O}} A$ 或者 $1 \otimes a \in \hat{\mathcal{K}} \otimes_{\mathcal{O}} A$ 都等同一致. 而且, 既然 $\mathcal{K} \cap \hat{\mathcal{O}} = \mathcal{O}$ 显然

2.15. As a general rule, since any $\mathcal{O}$-algebra A is an $\mathcal{O}$-free $\mathcal{O}$-module, we can identify $a \in A$ with $1 \otimes a \in \hat{\mathcal{O}} \otimes_{\mathcal{O}} A$, $1 \otimes a \in \mathcal{K} \otimes_{\mathcal{O}} A$ or $1 \otimes a \in \hat{\mathcal{K}} \otimes_{\mathcal{O}} A$. Moreover, since we have $\mathcal{K} \cap \hat{\mathcal{O}} = \mathcal{O}$, we clearly get

2.15.1 $$(\mathcal{K} \otimes_{\mathcal{O}} A) \cap (\hat{\mathcal{O}} \otimes_{\mathcal{O}} A) = A\,.$$

命题 2.16. 设 A 是一个 $\mathcal{O}$-代数. 如果每一个 $\mathcal{K} \otimes_{\mathcal{O}} A$ 的本原幂等元在 $\hat{\mathcal{K}} \otimes_{\mathcal{O}} A$ 里也是本原的, 那么每一个 A 的本原幂等元在 $\hat{\mathcal{O}} \otimes_{\mathcal{O}} A$ 里也是本原的.

Proposition 2.16. *Let A be an $\mathcal{O}$-algebra. If any primitive idempotent of $\mathcal{K} \otimes_{\mathcal{O}} A$ remains primitive in $\hat{\mathcal{K}} \otimes_{\mathcal{O}} A$, then any primitive idempotent of A still remains primitive in $\hat{\mathcal{O}} \otimes_{\mathcal{O}} A$.*

证明: 设 i 是一个 A 的本原幂等元; 只要用 iAi 代替 A 就能假定 A 的单元素是本原的. 那么, 设 $\hat{\imath}$ 是一个 $\hat{A} = \hat{\mathcal{O}} \otimes_{\mathcal{O}} A$ 的不等于零幂等元并且设 $\hat{J}'$ 和 $\hat{J}''$ 是两个 $\hat{\mathcal{K}} \otimes_{\hat{\mathcal{O}}} \hat{A}$ 的相互正交本原幂等元的集合使得

Proof: Let i be a primitive idempotent of A; up to the replacement of A by iAi, we may assume that the unity element of A is primitive. Then, let $\hat{\imath}$ be a nonzero idempotent of $\hat{A} = \hat{\mathcal{O}} \otimes_{\mathcal{O}} A$, and consider two sets $\hat{J}'$ and $\hat{J}''$ of pairwise orthogonal primitive idempotents such that

2.16.1 $$\sum_{\hat{\jmath} \in \hat{J}'} \hat{\jmath} = \hat{\imath}\,, \qquad \sum_{\hat{\jmath} \in \hat{J}''} \hat{\jmath} = 1 - \hat{\imath}\,.$$

另一方面, 由我们的假设, 一个满足 $\sum_{j\in J} j=1$ 的 $\mathcal{K}\otimes_{\mathcal{O}}A$ 的相互正交本原幂等元的集合 J 在 $\hat{\mathcal{K}}\otimes_{\mathcal{O}}A$ 里也是本原幂等元的集合. 从而, 由下面的引理 2.17, 存在一个双映射 $\tau\colon J\to\hat{J}$ 其中 $\hat{J}=\hat{J}'\cup\hat{J}''$, 和 $(\hat{\mathcal{K}}\otimes_{\mathcal{O}}A)^*$ 的元素 $\hat{a}$ 使得对任意 $j\in J$ 有 $\tau(j)=j^{\hat{a}}$; 令 $J'=(\tau)^{-1}(\hat{J}')$ 与 $i=\sum_{j\in J'} j$, 就特别有 $\hat{\imath}=i^{\hat{a}}$. 事实上, 我们能假定 $\hat{a}\in\hat{A}$; 那么选择 $h,\ell\in\mathbb{N}$ 使得

On the other hand, from our hypothesis, a set of pairwise orthogonal primitive idempotents of $\mathcal{K}\otimes_{\mathcal{O}}A$ such that $\sum_{j\in J} j=1$ remains a set of primitive idempotents in $\hat{\mathcal{K}}\otimes_{\mathcal{O}}A$. Moreover, according to Lemma 2.17 below, there are a bijective map $\tau\colon J\to\hat{J}$, where $\hat{J}=\hat{J}'\cup\hat{J}''$, and an element $\hat{a}$ in $(\hat{\mathcal{K}}\otimes_{\mathcal{O}}A)^*$ such that $\tau(j)=j^{\hat{a}}$ for any $j\in J$; in particular, setting $J'=(\tau)^{-1}(\hat{J}')$ and $i=\sum_{j\in J'} j$, we get $\hat{\imath}=i^{\hat{a}}$. Obviously, we may assume that $\hat{a}$ belongs to $\hat{A}$; then, consider $h,\ell\in\mathbb{N}$ such that

2.16.2 $$\hat{a}^{-1}\in\pi^{-h}\cdot\hat{A}\,,\quad i\in\pi^{-\ell}\cdot A\,.$$

设 $\{a_n\}_{n\in\mathbb{N}}$ 与 $\{b_n\}_{n\in\mathbb{N}}$ 是两个 A 的序列使得

Choose two sequences $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ of elements of A such that

2.16.3 $$\hat{a}=\lim_{n\to\infty}\{a_n\}\,,\quad \pi^h\cdot\hat{a}^{-1}=\lim_{n\to\infty}\{b_n\}\,;$$

也就是说, 对任意 $n\in\mathbb{N}$ 元素 $\hat{a}-a_n$ 和 $\pi^h\cdot\hat{a}^{-1}-b_n$ 都属于 $\pi^{n+1}\cdot\hat{A}$. 特别是, 使 $m\geq 2h+\ell$ 固定, 显然有

in other terms, for any $n\in\mathbb{N}$, the elements $\hat{a}-a_n$ and $\pi^h\cdot\hat{a}^{-1}-b_n$ belong to $\pi^{n+1}\cdot\hat{A}$. In particular, choosing $m\geq 2h+\ell$, we have

2.16.4 $$\begin{aligned}1&=\big(a_m+(\hat{a}-a_m)\big)\big(\pi^{-h}\cdot b_m+(\hat{a}^{-1}-\pi^{-h}\cdot b_m)\big)\\&=\pi^{-h}\cdot a_m b_m+c\end{aligned}$$

其中 c 属于 $\pi^{m+1-h}\cdot\hat{A}$, 还属于 $\mathcal{K}\otimes_{\mathcal{O}}A$, 这是因为 $a_m b_m$ 属于 A; 从而有

where c belongs to $\pi^{m+1-h}\cdot\hat{A}$ and, at the same time, belongs to $\mathcal{K}\otimes_{\mathcal{O}}A$ since $a_m b_m$ belongs to A; consequently, we have

2.16.5 $$c\in\pi^{m+1-h}\cdot\hat{A}\cap(\mathcal{K}\otimes_{\mathcal{O}}A)=\pi^{m+1-h}\cdot A$$

并且 A 的元素 $1-c$ 在 A 中是可逆的; 所以 a_m 在 $\mathcal{K}\otimes_{\mathcal{O}}A$ 中也是可逆的, 更精确有

and therefore the element $1-c$ is inversible in A; hence, a_m is inversible in $\mathcal{K}\otimes_{\mathcal{O}}A$ and explicitly we have

2.16.6 $$\begin{aligned}(a_m)^{-1}&=\pi^{-h}\cdot b_m(1-c)^{-1}\\&=\pi^{-h}\cdot b_m+\sum_{n=1}^{\infty}\pi^{-h}\cdot b_m c^n\,;\end{aligned}$$

这样$a_m\,i\,(a_m)^{-1}$属于$\mathcal{K}\otimes_{\mathcal{O}}A$，同时也属于$\hat{A}$，这是因为

thus, $a_m\,i\,(a_m)^{-1}$ belongs to $\mathcal{K}\otimes_{\mathcal{O}}A$ and, simultaneously, it belongs to $\hat{A}$ since

2.16.7 $$\hat{\imath}-a_m\,i\,(a_m)^{-1}=(\hat{a}-a_m)\,i\,(\hat{a}^{-1})+a_m\,i\,\big(\hat{a}^{-1}-(a_m)^{-1}\big)\,;$$

仍由于等式$\hat{A}\cap(\mathcal{K}\otimes_{\mathcal{O}}A)=A$，这个幂等元 $a_m\,i\,(a_m)^{-1}$ 属于A，从而它是单位元; 所以$\hat{\imath}$也是单位元.

hence, since $\hat{A}\cap(\mathcal{K}\otimes_{\mathcal{O}}A)=A$, the idempotent $a_m\,i\,(a_m)^{-1}$ belongs to A, and therefore it coincides with the unity element; thus, $\hat{\imath}$ coincides with unity element too.

引理 2.17. **设$\mathcal{L}$是一个域并设B是一个有限维数的$\mathcal{L}$-代数. 如果J与J'都是两个相互正交本原幂等元的集合使得$\sum_{j\in J}j=1=\sum_{j'\in J}j'$，那么存在$b\in B^*$使得$J'=J^b$.**

Lemma 2.17. *Let $\mathcal{L}$ be a field and B an $\mathcal{L}$-algebra of finite dimension. If J and J' are two sets of pairwise orthogonal primitive idempotents of B such that $\sum_{j\in J}j=1=\sum_{j'\in J}j'$, then there exists $b\in B^*$ such that $J'=J^b$.*

证明: 如果B的根基就是零那么, 只要使用 Wedderburn 定理, 就完成证明. 这样, 能假定B的根基非零, 那么有一个B的非零理想N使得$N^2=\{0\}$; 令$\bar{B}=B/N$，再令$\bar{b}$记$b\in B$的像; 首先指出, 如果i是B的本原幂等元, 那么$\bar{\imath}$也是本原的; 这是因为, 如果$0\neq\ell\in iBi$满足$\bar{\ell}^2=\bar{\ell}$，那么得到$0=(\ell^2-\ell)^2=\ell^4-2\ell^3+\ell^2$，而不难验证这个式子可推出

Proof: If the radical of B is zero, it suffices to apply the Wedderburn Theorem to prove the statement. Thus, we may assume that the radical of B is not zero, and then B has a nonzero ideal N such that $N^2=\{0\}$; set $\bar{B}=B/N$ and denote by $\bar{b}$ the image of $b\in B$; first of all, we claim that if i is a primitive idempotent of B then $\bar{\imath}$ still is primitive; indeed, if we choose $0\neq\ell\in iBi$ fulfilling $\bar{\ell}^2=\bar{\ell}$, then we get $0=(\ell^2-\ell)^2=\ell^4-2\ell^3+\ell^2$ according to our choice of N and, from this equality, it is not difficult to check that we have

2.17.1 $$\ell^n=(n-2)\ell^3-(n-3)\ell^2$$

其中$n\geq 2$，还可推出$3\ell^2-2\ell^3$是一个幂等元, 从而有$i=3\ell^2-2\ell^3$. 因为$\bar{\imath}=3\bar{\ell}^2-2\bar{\ell}^3=\bar{\ell}$所以$\bar{\imath}$也是本原的.

for any $n\geq 2$, which easily implies that $3\ell^2-2\ell^3$ is an idempotent and therefore we get $3\ell^2-2\ell^3=i$; since we have $\bar{\imath}=3\bar{\ell}^2-2\bar{\ell}^3=\bar{\ell}$, $\bar{\imath}$ is a primitive idempotent.

现在, 对维数$\dim_{\mathcal{L}}(B)$使用归纳法; 由上面的结果, $\bar{J}$与$\bar{J}'$是两个$\bar{B}$的相互正交本原幂等元的集合使得$\sum_{\bar{\jmath}\in\bar{J}}\bar{\jmath}=1=\sum_{\bar{\jmath}'\in\bar{J}}\bar{\jmath}'$; 而且, B^*显然包含$1+N$; 所以, 自然映射$B^*\to\bar{B}^*$是满射;

Now, we argue by induction on $\dim_{\mathcal{L}}(B)$; according to the previous argument, $\bar{J}$ and $\bar{J}'$ are two sets of pairwise orthogonal primitive idempotents of $\bar{B}$ such that $\sum_{\bar{\jmath}\in\bar{J}}\bar{\jmath}=1=\sum_{\bar{\jmath}'\in\bar{J}}\bar{\jmath}'$; moreover, B^* clearly contains $1+N$; consequently, the canonical map $B^*\to\bar{B}^*$ is surjective;

这样，存在 $b \in B^*$ 和一个双射 $\tau: J \to J'$ 使得对任意 $j \in J$ 有 $\overline{\tau(j)} = \overline{j^b}$. 那么，考虑 $c = \sum_{j \in J} jb\tau(j)$; 一方面可得到

thus, there are $b \in B^*$ and a bijective map $\tau: J \to J'$ such that, for any $j \in J$, we have $\overline{\tau(j)} = \overline{j^b}$. Then, consider the element $c = \sum_{j \in J} jb\tau(j)$; on the one hand, we have

2.17.2 $$\bar{c} = \sum_{j \in J} \bar{\jmath}\bar{b}\overline{j^b} = \bar{b},$$

从而 c 是可逆的; 另一方面对任意 $j \in J$ 有 $jc = jb\tau(j) = c\tau(j)$.

and therefore c is inversible; on the other hand, we have $jc = jb\tau(j) = c\tau(j)$ for any $j \in J$.

推论 2.18. 设 A 是 $\mathcal{O}$-代数. 令 J 记 A 的根基并 $s: A \to A/J = \bar{A}$ 记自然的映射. 假定 $\mathcal{K} \otimes_{\mathcal{O}} A$ 的每一个本原幂等元在 $\hat{\mathcal{K}} \otimes_{\mathcal{O}} A$ 里也是本原的. 那么对任意 $\bar{A}$ 的幂等元 $\bar{\imath}$ 存在一个幂等元 $i \in A$ 使得 $s(i) = \bar{\imath}$. 而且, 如果 $i' \in A$ 也是一个幂等元使得 $s(i') = \bar{\imath}$, 就存在 $a \in A^*$ 使得 $i' = i^a$

Corollary 2.18. *Let A be an $\mathcal{O}$-algebra. Denote by J the radical of A and by $s: A \to \bar{A} = A/J$ the canonical map. Assume that any primitive idempotent of $\mathcal{K} \otimes_{\mathcal{O}} A$ remains primitive in $\hat{\mathcal{K}} \otimes_{\mathcal{O}} A$. Then, for any idempotent $\bar{\imath}$ of $\bar{A}$, there is an idempotent $i \in A$ such that $s(i) = \bar{\imath}$. Moreover, if $i' \in A$ is also an idempotent such that $s(i') = \bar{\imath}$, then there is $a \in A^*$ such that $i' = i^a$.*

证明: 设 J 是一个 A 的相互正交本原幂等元的集合使得 $\sum_{j \in J} j = 1$ 并且设 $\bar{J}'$ 和 $\bar{J}''$ 是两个 $\bar{A}$ 的相互正交本原幂等元的集合使得

Proof: Let J be a set of pairwise orthogonal primitive idempotents of A such that $\sum_{j \in J} j = 1$, and choose two sets $\bar{J}'$ and $\bar{J}''$ of pairwise orthogonal primitive idempotents of $\bar{A}$ such that

2.18.1 $$\sum_{\bar{\jmath} \in \bar{J}'} \bar{\jmath} = \bar{\imath}, \quad \sum_{\bar{\jmath} \in \bar{J}''} \bar{\jmath} = 1 - \bar{\imath}.$$

由命题 2.16, 对任意 $j \in J$, 幂等元 $s(j)$ 在 $\bar{A}$ 里也是本原的; 那么, 由引理 2.17, 存在双映射 $\tau: J \to \bar{J}' \cup \bar{J}''$ 和 $\bar{a} \in \bar{A}^*$ 使得对任意 $j \in J$ 有 $\tau(j) = s(j)^{\bar{a}}$. 可是, 任意满足 $s(a) = \bar{a}$ 的元素 $a \in A$ 是可逆的, 这是因为, 由 Nakayama 引理, 从 $\bar{A}\bar{a} = \bar{A}$ 可推出 $Aa = A$. 所以有

According to Proposition 2.16, the idempotent $s(j)$ is also primitive in $\bar{A}$ for any $j \in J$; then, according to Lemma 2.17, there are an element $\bar{a} \in \bar{A}^*$ and a bijective map $\tau: J \to \bar{J}' \cup \bar{J}''$ such that we have $\tau(j) = s(j)^{\bar{a}}$ for any $j \in J$. But any element $a \in A$ such that $s(a) = \bar{a}$ is inversible since, according to the Nakayama Lemma, from $\bar{A}\bar{a} = \bar{A}$, we can deduce that $Aa = A$. Hence, we have

2.18.2 $$s\Big(\sum_{j \in (\tau)^{-1}(\bar{J}')} j^a\Big) = \bar{\imath}.$$

最后, 如果 i 与 i' 是两个 A 的幂等元使得 $s(i)=\bar{\imath}=s(i')$, 那么一方面 $c=ii'+(1-i)(1-i')$ 也是可逆的, 因为从 $s(c)=1$ 可推出 $Ac=A$; 另一方面, 显然有 $ic=ii'=ci'$.

Finally, if i and i' are two idempotents of A such that $s(i)=\bar{\imath}=s(i')$ then, on the one hand, $c=ii'+(1-i)(1-i')$ is inversible since $s(c)=1$ implies that $Ac=A$; on the other hand, clearly we have $ic=ii'=ci'$.

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