CLASSIFICATION DES ALGÈBRES DE LIE SIMPLES RÉELLES

Algèbres de lie à structure complexe et algèbres de lie compactes (cf HELGASON 1978 p 516)

<u>Isomorphismes</u> $\mathfrak{so}(3,\mathbb{C}) \simeq \mathfrak{sl}(2,\mathbb{C})$; $\mathfrak{sp}(2,\mathbb{C}) \simeq \mathfrak{sl}(2,\mathbb{C})$; $\mathfrak{sp}(4,\mathbb{C}) \simeq \mathfrak{so}(5,\mathbb{C})$; $\mathfrak{so}(6,\mathbb{C}) \simeq \mathfrak{sl}(4,\mathbb{C})$; [et $\mathfrak{so}(4,\mathbb{C}) \simeq \mathfrak{sl}(2,\mathbb{C}) \times \mathfrak{sl}(2,\mathbb{C})$]
$\mathfrak{so}(3) \simeq \mathfrak{su}(2)$; $\mathfrak{sp}(1) \simeq \mathfrak{su}(2)$; $\mathfrak{sp}(2) \simeq \mathfrak{so}(5)$; $\mathfrak{so}(6) \simeq \mathfrak{su}(4)$; [et $\mathfrak{so}(4) \simeq \mathfrak{su}(2) \times \mathfrak{su}(2)$]

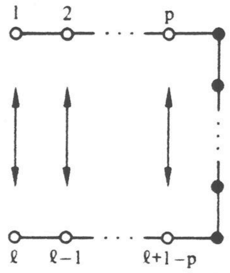
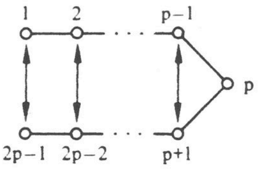
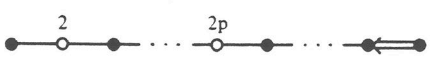
$\mathfrak{g}$ type d'alg lie simple/$\mathbb{C}$	G gr lie connexe d'alg lie $\mathfrak{g}$	U sous-gr compact max de G	$Z(\tilde{U})$ centre du rev univ de U	dim U
$\mathfrak{a}_n (n \geq 1)$	$SL(n+1, C)$	$SU(n+1)$	$\mathbb{Z}/(n+1)\mathbb{Z}$	$n(n+2)$
$\mathfrak{b}_n (n \geq 1)$	$SO(2n+1, C)$	$SO(2n+1)$	$\mathbb{Z}/2\mathbb{Z}$	$n(2n+1)$
$\mathfrak{c}_n (n \geq 1)$	$Sp(2n, C)$	$Sp(n)$	$\mathbb{Z}/2\mathbb{Z}$	$n(2n+1)$
$\mathfrak{d}_n (n \geq 3)$	$SO(2n, C)$	$SO(2n)$	$\mathbb{Z}/4\mathbb{Z}$ si n impair $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ si n pair	$n(2n-1)$
$\mathfrak{e}_6$	E_6^C	E_6	$\mathbb{Z}/3\mathbb{Z}$	78
$\mathfrak{e}_7$	E_7^C	E_7	$\mathbb{Z}/2\mathbb{Z}$	133
$\mathfrak{e}_8$	E_8^C	E_8	$\{1\}$	248
$\mathfrak{f}_4$	F_4^C	F_4	$\{1\}$	52
$\mathfrak{g}_2$	G_2^C	G_2	$\{1\}$	14

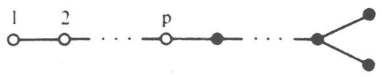
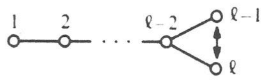
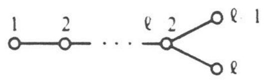
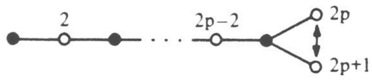
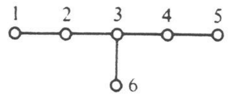
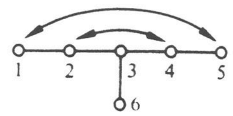
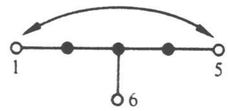
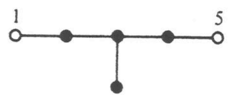
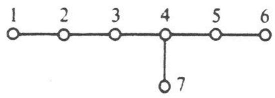
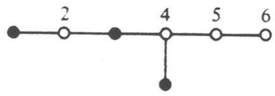
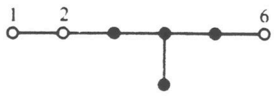
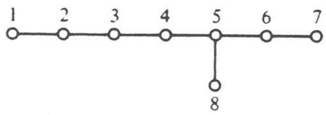
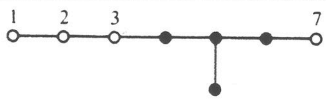
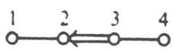
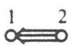
De plus $\mathfrak{g} \simeq \mathfrak{u}_{\mathbb{C}}$ et :

- un groupe de lie réel semi-simple connexe a même centre et même groupe fondamental qu'un de ses sous-groupes compacts maximaux ;
- une algèbre de lie de dim finie sur $\mathbb{C}$ a même dimension (sur $\mathbb{C}$) qu'une de ses formes réelles (sur $\mathbb{R}$).

Autres algèbres de lie simples réelles (cf ONISHCHIK et VINBERG 1990 p 312 à p 317)

<u>Isomorphismes</u> $\mathfrak{su}(1,1) \simeq \mathfrak{sl}(2,\mathbb{R})$; $\mathfrak{so}(1,2) \simeq \mathfrak{sl}(2,\mathbb{R})$; $\mathfrak{sp}(2,\mathbb{R}) \simeq \mathfrak{sl}(2,\mathbb{R})$, $\mathfrak{sp}(1,1) \simeq \mathfrak{so}(1,4)$; $\mathfrak{sp}(4,\mathbb{R}) \simeq \mathfrak{so}(2,3)$;
$\mathfrak{so}^*(8) \simeq \mathfrak{so}(2,6)$; $\mathfrak{so}(3,3) \simeq \mathfrak{sl}(4,\mathbb{R})$, $\mathfrak{so}(1,5) \simeq \mathfrak{su}^*(4)$, $\mathfrak{so}^*(6) \simeq \mathfrak{su}(1,3)$, $\mathfrak{so}(2,4) \simeq \mathfrak{su}(2,2)$;
[et $\mathfrak{so}(1,3) \simeq \mathfrak{sl}(2,\mathbb{C})|_{\mathbb{R}}$, $\mathfrak{so}(2,2) \simeq \mathfrak{sl}(2,\mathbb{R}) \times \mathfrak{sl}(2,\mathbb{R})$, $\mathfrak{so}^*(4) \simeq \mathfrak{su}(2) \times \mathfrak{sl}(2,\mathbb{R})$]

$\mathfrak{g}(\mathbb{C})$	$\mathfrak{g}$	$\mathfrak{k}$	Satake diagram	Type of Σ	r	dim $\mathfrak{g}_{\lambda_j}$	dim $\mathfrak{g}_{2\lambda_j}$
$\mathfrak{sl}_{l+1}(\mathbb{C})$ $(l \geq 1)$	$\mathfrak{sl}_{l+1}(\mathbb{R})$	$\mathfrak{so}_{l+1}$		A_ℓ	$r(\alpha_j) = \lambda_j$ $(1 \leq j \leq l)$	1	0
	$\mathfrak{su}^*_{2p+2}$ $(l = 2p+1, p \geq 1)$	$\mathfrak{sp}_{p+1}$		A_p	$r(\alpha_{2j}) = \lambda_j$ $(1 \leq j \leq p)$	4	0
	$\mathfrak{su}_{p,l+1-p}$ $(1 \leq p \leq \ell/2)$	$\mathfrak{su}_p \oplus \mathfrak{u}_{l+1-p}$		BC_p	$r(\alpha_j) = r(\alpha_{l+1-j})$ $= \lambda_j$ $(1 \leq j \leq p)$	$2\ (j \leq p+1)$	0
						$2(l+1-2p)\ (j = p)$	1
	$\mathfrak{su}_{p,p}$ $(l = 2p-1, p \geq 2)$	$\mathfrak{su}_p \oplus \mathfrak{u}_p$		C_p	$r(\alpha_j) = r(\alpha_{2p-j})$ $= \lambda_j$ $(1 \leq j \leq p)$	$2\ (j \leq p-1)$ $1\ (j = p)$	0
$\mathfrak{so}_{2l+1}(\mathbb{C})$ $(l \geq 1)$	$\mathfrak{so}_{p,2l+1-p}$ $(1 \leq p \leq l)$	$\mathfrak{so}_p \oplus \mathfrak{so}_{2l+1-p}$		B_p	$r(\alpha_j) = \lambda_j$ $(1 \leq j \leq p)$	$1\ (j \leq p-1)$ $2(l-p)+1\ (j = p)$	0
$\mathfrak{sp}_{2l}(\mathbb{C})$ $(\ell \geq 1)$	$\mathfrak{sp}_{2l}(\mathbb{R})$	$\mathfrak{u}_l$		C_ℓ	$r(\alpha_j) = \lambda_j$ $(1 \leq j \leq l)$	1	0
	$\mathfrak{sp}_{p,l-p}$ $(1 \leq p < \frac{1}{2}(l-1))$	$\mathfrak{sp}_p \oplus \mathfrak{sp}_{l-p}$		BC_p	$r(\alpha_{2j}) = \lambda_j$ $(1 \leq j \leq p)$	$4\ (j \leq p-1)$	0
						$4(l-2p)\ (j = p)$	3
	$\mathfrak{sp}_{p,p}$ $(l = 2p)$	$\mathfrak{sp}_p \oplus \mathfrak{sp}_p$		C_p	$r(\alpha_{2j}) = \lambda_j$ $(1 \leq j \leq p)$	$4\ (j \leq p-1)$ $3\ (j = p)$	0

$\mathfrak{g}(\mathbb{C})$	$\mathfrak{g}$	$\mathfrak{k}$	Satake diagram	Type of Σ	r	$\dim \mathfrak{g}_{\lambda_j}$	$\dim \mathfrak{g}_{2\lambda_j}$
$\mathfrak{so}_{2l}(\mathbb{C})$ $(l \geq 3)$	$\mathfrak{so}_{p,2l-p}$ $(1 \leq p \leq l-2)$	$\mathfrak{so}_p \oplus \mathfrak{so}_{2l-p}$		B_p	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq p)$	$1\ (j \leq p-1)$ $2(l-p)\ (j=p)$	0
	$\mathfrak{so}_{l-1,l+1}$	$\mathfrak{so}_{l-1} \oplus \mathfrak{so}_{l+1}$		B_{l-1}	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq l-1)$, $r(\alpha_l)=\lambda_{l-1}$	$1\ (j \leq l-2)$ $2\ (j=l-1)$	0
	$\mathfrak{so}_{l,l}$	$\mathfrak{so}_l \oplus \mathfrak{so}_l$		D_l	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq l)$	1	0
	$\mathfrak{so}^*_{4p}$ $(l=2p)$	$\mathfrak{u}_{2p}$		C_p	$r(\alpha_{2j})=\lambda_j$ $(1 \leq j \leq p)$	$4\ (j \leq p-1)$ $1\ (j=p)$	0
	$\mathfrak{so}^*_{4p+2}$ $(l=2p+1)$	$\mathfrak{u}_{2p+1}$		BC_p	$r(\alpha_{2j})=\lambda_j$ $(1 \leq j \leq p)$ $r(\alpha_{2p+1})=\lambda_p$	4	$0\ (j \leq p-1)$ $1\ (j=p)$
E_6	EI	$\mathfrak{sp}_4$		E_6	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq 6)$	1	0
	EII	$\mathfrak{su}_2 \oplus \mathfrak{su}_6$		F_4	$r(\alpha_1)=r(\alpha_5)=\lambda_1$, $r(\alpha_2)=r(\alpha_4)=\lambda_2$, $r(\alpha_3)=\lambda_3$, $r(\alpha_6)=\lambda_4$	$2\ (j=1,2)$ $1\ (j=3,4)$	0
	$EIII$	$\mathfrak{so}_{10} \oplus \mathbf{R}$		BC_2	$r(\alpha_1)=r(\alpha_5)=\lambda_2$, $r(\alpha_6)=\lambda_1$	$6\ (j=1)$ $8\ (j=2)$	0 1
	EIV	F_4		A_2	$r(\alpha_1)=\lambda_1$, $r(\alpha_5)=\lambda_2$	8	0
E_7	EV	$\mathfrak{su}_8$		E_7	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq 7)$	1	0
	EVI	$\mathfrak{su}_2 \oplus \mathfrak{so}_{12}$		F_4	$r(\alpha_2)=\lambda_1$, $r(\alpha_4)=\lambda_2$, $r(\alpha_5)=\lambda_3$, $r(\alpha_6)=\lambda_4$	$4\ (j=1,2)$ $1\ (j=3,4)$	0
	$EVII$	$E_6 \oplus \mathbf{R}$		C_3	$r(\alpha_6)=\lambda_1$, $r(\alpha_2)=\lambda_2$, $r(\alpha_1)=\lambda_3$	$8\ (j=1,2)$ $1\ (i=3)$	0
E_8	$EVIII$	$\mathfrak{so}_{16}$		E_8	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq 8)$	1	0
	EIX	$\mathfrak{su}_2 \oplus E_7$		F_4	$r(\alpha_7)=\lambda_1$, $r(\alpha_3)=\lambda_2$, $r(\alpha_2)=\lambda_3$, $r(\alpha_1)=\lambda_4$	$8\ (j=1,2)$ $1\ (j=3,4)$	0
F_4	FI	$\mathfrak{su}_2 \oplus \mathfrak{sp}_3$		F_4	$r(\alpha_j)=\lambda_j$ $(1 \leq j \leq 4)$	1	0
	FII	$\mathfrak{so}_9$		BC_1	$r(\alpha_1)=\lambda_1$	8	7
G_2	G	$\mathfrak{so}_3 \oplus \mathfrak{so}_3$		G_2	$r(\alpha_j)=\lambda_j$ $(j=1,2)$	1	0