« Formules du caractère et intégrales orbitales sur un groupe réductif réel non connexe » (J.-Y. Ducloux, Université Paris 7, May 23, 2002)

Let G be a real reductive Lie group and s be a semisimple element in G. The problem here is to write the orbital integral of s in terms of the characters $\operatorname{tr}\pi$ of G. Such a formula has been obtained by R. Herb in '83 in the case when G is connected, and by D. Renard in '97 when G/G_0 is cyclic. From the work of M. Duflo (concrete Plancherel formula for real linear algebraic groups in '83) and also of M. Vergne and Khalgui-Torasso (Poisson-Plancherel formula in '97), one can expect to obtain general inversion formulas using the orbit method. With this point of view, the "descent method" allows us to suppose that s is elliptic. Then we'll have:

1. to parametrize in terms of coadjoint orbits some subset of the unitary dual of G, and to find some "Kirillov's formulas" for $\operatorname{tr}\pi(e\exp X)$, when π is in the subset mentioned above, e is an elliptic element of G and X is an element close to 0 of the Lie algebra of the commutant $G(e)$ of e in G (cf. my paper in the Journal of Lie Theory, '02);

2. to construct a discrete set of invariant integrals on the Lie algebra of $G(e)$ with some associated signs and write a conjecture ("Poisson-orbital formula") for the sum of the product of these signs with the corresponding invariant integral, such that it formally solves our problem.