Errata on "Representations of finite groups of Lie type"

- In the table of contents, chapter 11: "Mackey formula" instead of "Mackay formula"
- page 6, line 3: "irreducible" instead of "closed".
- page 9, line 10: at the end of the line add "if $\mathbf{G}$ is connected"
- page 17, proposition 0.43 "semi-simple" instead of "reductive".
- page 20, line -5: $BwB \cup BswB$ instead of $BsB \cup BswB$.
- Page 23, line -1: "set" instead of "group".
- page 24, line 10: expand $\mathbf{L}_I \cap \mathbf{V}_I = 1$ into: by 0.33, $\mathbf{L}_I \cap \mathbf{V}_I$ contains no $\mathbf{U}_\alpha$ thus $\mathbf{L}_I \cap \mathbf{V}_I = 1$
- page 36, line -9: "for all generators of A, so that $F'^n(x) = F^n(x)$ for all $x \in A$" instead of "for all $x \in A$"
- page 37, line 21: "whose square" instead of "whose square or cube".
- page 37, Exercise 3.8: add "up to conjugation by an automorphism of $\mathbb{A}^1$"
- page 37, replace lines -9 to -3 by:

For classical groups, that is algebraic groups such as the linear, orthogonal or symplectic groups which are defined as groups of matrices (see chapter 15), we define the **standard** Frobenius endomorphism as the restriction to $\mathbf{G}$ of the endomorphism of GL_n defined by $T_{ij} \mapsto T_{ij}^q$. There are other rational structures on such groups; for instance the unitary group is $\mathrm{GL}_n^{F'}$ where F' is the Frobenius endomorphism defined by $F'(x) = F({}^t x^{-1})$, with F being the standard Frobenius endomorphism on GL_n.

Remark of the authors: the tentative to define a priori the "standard" rational structure on any algebraic group by an embedding into GL_n*, e.g. chosen of minimal dimension and with image stable by* $T_{ij} \mapsto T_{ij}^q$*, is doomed to failure since there are usually two such embeddings (giving for instance the two rational structures on* GL_n*). On the other hand, one could define standard as the existence of a maximal torus* $\mathbf{T}$ *such that* F *acts on* $X(\mathbf{T})$ *by multiplication by* q*.*

- page 39, line 12: $\mathrm{PSL}_n^F / (\mathrm{SL}_n^F / \mu_n^F)$ instead of $(\mathrm{SL}_n / \mu_n)^F$.
- page 39, 3.15(iii), (v) and (vi): assume $\mathbf{G}$ reductive.
- page 40, line -16: "over $\overline{\mathbb{F}}_q$" instead of "in $\overline{\mathbb{F}}_q$".
- page 40, line -13: replace "it is clear" by "it can be proved (see [Sp, 11.4.7], 9.6.3 in the second edition)"

- page 49, just before NOTATION, add the following paragraph:

Note that $R_{\mathbf{L}}^{\mathbf{G}}$ can also be described as the natural lifting from $\mathbf{L}^F$ to $\mathbf{P}^F$ followed with induction from $\mathbf{P}^F$ to $\mathbf{G}^F$; similarly ${}^*R_{\mathbf{L}}^{\mathbf{G}}$ is restriction from $\mathbf{G}^F$ to $\mathbf{P}^F$ followed with the taking of fixed points under $\mathbf{U}^F$.

- page 52, line 1: $l(v)+l(w)>l(vw)$ instead of $l(v)+l(w)<l(vw)$.
- page 61 line -2: $\mathcal{C}(\mathbf{L}^F)$ instead of $\mathcal{C}(\mathbf{G}^F)$.
- page 62 line 2: $l\in\mathbf{L}^F$ instead of $l\in\mathbf{L}$.
- page 67 line -1: "$= {}^x\mathbf{M}$ (equality by 1.18)" instead of "$= {}^x\mathbf{M}$".
- page 68 line -13: add "rational" before "maximal".
- page 83 line 2: "$\mathbf{G}^F$-varieties-$\mathbf{M}^F$" instead of "$\mathbf{L}^F$-varieties-$\mathbf{M}^F$".
- page 83, line -6: $(x,x')=(\gamma y,\gamma y')$ instead of $(x,x')=(\gamma x,\gamma x')$.
- page 86, line -13: ${}^{w'}Z(\mathbf{L})^0\subset Z(\mathbf{M}^0)$ instead of ${}^{w'}Z(\mathbf{L})^0 = Z(\mathbf{M}^0)$.
- page 89, line 8: $\mathbf{T}_{w'}$ instead of $bT_{w'}$.
- page 90, line 12: suppress the (false) sentence "The values of the Green functions are in $\mathbb{Z}$ by 10.6."
- page 96, lines 6 and 7: replace "By 7.4 and 7.5 $\ldots\gamma_p$" with "as seen in the proof of 9.4 $\mathrm{reg}_{\mathbf{G}}=\mathrm{St}_{\mathbf{G}}\,\gamma_p$".
- page 97, line 7: "the Mackey formula 11.13" instead of "the Mackey formula 11.12".
- page 98, line -6: "which is $|C_{\mathbf{G}}(s)^F/C^0_{\mathbf{G}}(s)^F|$ times" instead of "which is equal to"
- page 98, line -5: In the formula replace (twice) "$\varepsilon_{\mathbf{G}}$" with "$\varepsilon_{C^\circ_{\mathbf{G}}(s)}$".
- pages 100–101: replace the beginning of the proof of 13.3 and 13.4 by:

PROOF: Let χ be the common irreducible constituent of the statement; we may assume that χ is a component of $R_{\mathbf{T}}^{\mathbf{G}}(\theta)$ and we shall show that $(\mathbf{T},\theta)$ and $(\mathbf{T}',\theta')$ are geometrically conjugate. We remark first that by 10.6 χ occurring in $R_{\mathbf{T}}^{\mathbf{G}}(\theta)$ is equivalent to $\overline{\chi}$ occurring in $R_{\mathbf{T}}^{\mathbf{G}}(\overline{\theta})$, which implies that $\overline{\chi}^\vee$ occurs in $\overline{\theta}^\vee\otimes H^i_c(\mathcal{L}^{-1}(\mathbf{U})^\vee)$ for some k (where, given a left representation χ of a group H, we let $\chi^\vee$ denote the right representation obtained by making elements act through their inverse). As χ occurs in $H^j_c(\mathcal{L}^{-1}(\mathbf{U}'))\otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{T}'^F]}\theta'$, the representation $\overline{\theta}\otimes\theta'^\vee$ of $\mathbf{T}^F\times\mathbf{T}'^F$ occurs in the module $H^i_c(\mathcal{L}^{-1}(\mathbf{U})^\vee)\otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{G}^F]}H^j_c(\mathcal{L}^{-1}(\mathbf{U}'))$, which with the notation of 11.7 is a submodule of $H^{i+j}_c(\mathbf{Z})$. However, we have:

13.4 LEMMA. *If the $\mathbf{T}^F$-module-${}^w(\mathbf{T}'^F)$ given by $\overline{\theta}\otimes{}^w\theta'^\vee$ occurs in some cohomology group of $\mathbf{Z}''_w$ (see 11.8) and if $n>0$ is such that ${}^{F^n}w=w$, then* $\theta\circ N_{F^n/F}=\theta'\circ N_{F^n/F}\circ\mathrm{ad}\,{}^Fw^{-1}$.

- page 106, line 19: “Let $\mathbf{T}$” instead of “Let $\mathbf{G}$”.
- page 107, lines 15 and 17: $\langle\Phi\rangle$ instead of Φ.
- page 109, lines 2–3: Delete “which may be written…” and replace “Let $t \in \mathbf{T}$ be such that $t^{-1}\,{}^F t = z$” by “Let $t \in \mathbf{T}$ be such that $t \cdot {}^F t^{-1} = z$.
- page 110, line 4: “F-stable Levi subgroup of some parabolic subgroup” instead of “F-stable Levi subgroup of some F-stable parabolic subgroup”.
- page 110, line -7: in equation (2), $f(g)$ should be g.
- page 112, Theorem 13.23: replace $\mathcal{E}(\mathbf{G}^F, (s))$ with the rational Lusztig series $\mathcal{E}(\mathbf{G}^F, (s)_{\mathbf{G}^{*F^*}})$ as defined page 136 above 14.41.
- page 113, last line of the proof of 13.24: one cannot apply (1) directly, since $C_{\mathbf{G}^*}(s)^{F^*}$ is always connected, but in the formula

$$\frac{\langle\, \psi_s(\chi), R_{\mathbf{T}^*}^{C_{\mathbf{G}^*}(s)}(\mathrm{Id}_{\mathbf{T}^*})\,\rangle_{C_{\mathbf{G}^*}(s)^{F^*}}}{\langle\, R_{\mathbf{T}^*}^{C_{\mathbf{G}^*}(s)}(\mathrm{Id}_{\mathbf{T}^*}), R_{\mathbf{T}^*}^{C_{\mathbf{G}^*}(s)}(\mathrm{Id}_{\mathbf{T}^*})\,\rangle_{C_{\mathbf{G}^*}(s)^{F^*}}} R_{\mathbf{T}^*}^{C_{\mathbf{G}^*}(s)}(\mathrm{Id}_{\mathbf{T}^*})(1)$$

  the denominator is equal to $|C_{\mathbf{G}^*}(s)^{F^*}/C^\circ_{\mathbf{G}^*}(s)^{F^*}|$ times the analogous expression in the connected component $C^\circ_{\mathbf{G}^*}(s)$ and $R_{\mathbf{T}^*}^{C_{\mathbf{G}^*}(s)}(\mathrm{Id}_{\mathbf{T}^*})(1)$ is equal to the same coefficient times $R_{\mathbf{T}^*}^{C^\circ_{\mathbf{G}^*}(s)}(\mathrm{Id}_{\mathbf{T}^*})(1)$. Using then Frobenius reciprocity in the numerator we get the same expression in the connected centralizer $C^\circ_{\mathbf{G}^*}(s)$, with $\psi_s(\chi)$ replaced with its restriction to $C^\circ_{\mathbf{G}^*}(s)$. We can now apply (1) in $C^\circ_{\mathbf{G}^*}(s)$ and get the result since $\psi_s(\chi)$ and its restriction have same dimension.
- page 114: Replace (i) in the statement of theorem 13.25 with “ For any $\pi \in \mathcal{E}(\mathbf{L}^F, (s))$ there exists an integer $i(\pi)$ such that the space $H^i_c(\mathcal{L}^{-1}(\mathbf{U})) \otimes_{\overline{\mathbb{Q}}_\ell} \pi$ is zero for $i \neq i(\pi)$ and affords an irreducible representation of $\mathbf{G}^F$ for $i = i(\pi)$.”
- page 114, line -14: $W_{\mathbf{L}^*}(\mathbf{T}_w)$ should be $W_{\mathbf{L}^*}(\mathbf{T}^*_w)$.
- page 114, line -1: $\mathbf{P}'$ instead of $\mathbf{Q}$.
- page 115, line 6 and page 116, lines 1 and 4: $\langle \pi, \overline{\pi}'\rangle_{\mathbf{L}^F}$ instead of $\langle \pi, \pi'\rangle_{\mathbf{L}^F}$.
- page 116 line 4: H^* should be H^*_c (twice).
- page 116: replace the paragraph which begins by “We now prove theorem 13.25.” by

We now prove theorem 13.25. From 13.27 the dimension of

$$\oplus_{i+j=2d} \pi^\vee \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H^i_c(\mathcal{L}^{-1}(\mathbf{U})^\vee) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H^j_c(\mathcal{L}^{-1}(\mathbf{U})^\vee) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi \simeq \pi^\vee \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H^i_c(\mathbf{Z}) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi$$

is equal to 1, so all the summands have dimension 0 except one, say

$$\pi^\vee \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H_c^{i(\pi)}(\mathcal{L}^{-1}(\mathbf{U})^\vee) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H_c^{2d-i(\pi)}(\mathcal{L}^{-1}(\mathbf{U})^\vee) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi$$

which has dimension 1. Suppose that the $\mathbf{G}^F$-module given by

$$H_c^j(\mathcal{L}^{-1}(\mathbf{U})) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi$$

is not 0. Let χ be one of its irreducible components. Then χ is in $\mathcal{E}(\mathbf{G}^F,(s))$, and by 13.26 (ii), it is a component of $R_{\mathbf{T}}^{\mathbf{G}}(\theta)$ where $\mathbf{T}$ is a maximal torus of $\mathbf{L}$ and θ is given by the geometric class of s, so $\overline{\chi}$ is a component of $R_{\mathbf{T}}^{\mathbf{G}}(\overline{\theta})$. So $\overline{\chi}$ is a component of $R_{\mathbf{L}}^{\mathbf{G}} R_{\mathbf{T}}^{\mathbf{L}}(\overline{\theta})$ and in particular appears in some $R_{\mathbf{L}}^{\mathbf{G}}(\overline{\pi}')$ with $\pi' \in \mathcal{E}(\mathbf{L}^F,(s))$. Then $\overline{\chi}^\vee$ occurs in some $\pi^\vee \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H_c^k(\mathcal{L}^{-1}(\mathbf{U})^\vee)$ and thus

$$\pi^\vee \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H_c^j(\mathcal{L}^{-1}(\mathbf{U})^\vee) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{G}^F]} H_c^k(\mathcal{L}^{-1}(\mathbf{U})) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi \neq 0.$$

But this is a subspace of $\pi^\vee \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} H_c^{j+k}(\mathbf{Z},\overline{\mathbb{Q}}_\ell) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi$, so this last space is not 0, which proves by 13.27 that $j = i(\pi)$. Since in that case the last space is of dimension at most 1, we see that χ must be in addition the only irreducible component of $H_c^{i(\pi)}(\mathcal{L}^{-1}(\mathbf{U})) \otimes_{\overline{\mathbb{Q}}_\ell[\mathbf{L}^F]} \pi$. Whence (i) of the theorem.

- page 116 line -6: $R_{\mathbf{T}}^{\mathbf{G}}(\theta')$ should be $R_{\mathbf{T}'}^{\mathbf{G}}(\theta')$
- pages 118, replace the proof of Proposition 13.30 (i) by the following (we thank Radha Kessar for noticing the problem):

Proof. We follow [DL1, 5.11].

As $s \in Z(\mathbf{G}^{*F^*})$, its geometric class defines a character of the rational points of any rational maximal torus of $\mathbf{G}$ since, if $(\mathbf{G},\mathbf{T})$ is dual to $(\mathbf{G}^*,\mathbf{T}^*)$, the element s is in $\mathbf{T}^{*wF^*}$ for any w. Let $(\tilde{\mathbf{G}},\tilde{\mathbf{T}})$ be a dual pair to $(\mathbf{G}^*/Z(\mathbf{G}^*),\mathbf{T}^*/Z(\mathbf{G}^*))$. The following can be shown:

Proposition. *The quotient morphism* $\mathbf{G}^* \xrightarrow{\pi^*} \mathbf{G}^*/Z(\mathbf{G}^*)$ *corresponds to an* F*-equivariant morphism* $\tilde{\mathbf{G}} \xrightarrow{\pi} \mathbf{G}$ *with central kernel, which induces an isomorphism on the* $\mathbf{U}_\alpha$ *and such that* $\pi(\tilde{\mathbf{T}}) \subset \mathbf{T}$.

It follows from the isomorphism on the $\mathbf{U}_\alpha$ that $\pi(\tilde{\mathbf{G}}') = \mathbf{G}'$, where $'$ denotes the derived subgroup. Moreover $\pi(\tilde{\mathbf{T}} \cap \tilde{\mathbf{G}}')$, being a maximal torus of $\pi(\tilde{\mathbf{G}}') = \mathbf{G}'$ contained in $\mathbf{T} \cap \mathbf{G}'$, has to be equal to $\mathbf{T} \cap \mathbf{G}'$. We thus get $\pi(Z(\tilde{\mathbf{G}}')) = Z(\mathbf{G}')$ since the center consists of the elements in a maximal torus which act trivially on the $\mathbf{U}_\alpha$. Since the kernel of π is central, we deduce that π induces an isomorphism $(\tilde{\mathbf{T}} \cap \tilde{\mathbf{G}}')/Z(\tilde{\mathbf{G}}') \simeq (\mathbf{T} \cap \mathbf{G}')/Z(\mathbf{G}')$.

Lemma. $\mathbf{T} \cap \pi(\tilde{\mathbf{G}}) = \pi(\tilde{\mathbf{T}})$ *and* $\mathbf{T}^F \cap \pi(\tilde{\mathbf{G}}^F) = \pi(\tilde{\mathbf{T}}^F)$.

Proof. Using 0.40 we can write any element of $\tilde{\mathbf{G}}$ as $\tilde{t}\tilde{g}$ with $\tilde{t} \in \tilde{\mathbf{T}}$ and $\tilde{g} \in \tilde{\mathbf{G}}'$. If $\pi(\tilde{t}\tilde{g}) \in \mathbf{T}$, since $\pi(\tilde{g}) \in \mathbf{G}'$ we have $\pi(\tilde{g}) \in \mathbf{T} \cap \mathbf{G}' \subset \pi(\tilde{\mathbf{T}})$. Since the kernel of π is contained in $\tilde{\mathbf{T}}$ we get $\tilde{g} \in \tilde{\mathbf{T}}$. If moreover we start with an element of $\tilde{\mathbf{G}}^F$, since $\tilde{\mathbf{G}}' \cap \tilde{\mathbf{T}}$ is connected we can assume that $\tilde{t}$ and $\tilde{g}$ are F-fixed in the above computation and we get $\tilde{g} \in \tilde{\mathbf{T}}^F$. $\square$

Lemma. *The inclusion* $\mathbf{T} \subset \mathbf{G}$ *induces an isomorphism* $\mathbf{T}^F/\pi(\tilde{\mathbf{T}}^F) \simeq \mathbf{G}^F/\pi(\tilde{\mathbf{G}}^F)$.

Proof. We have $\mathbf{G} = \mathbf{T}\mathbf{G}' = \mathbf{T}\pi(\tilde{\mathbf{G}})$, the first equality by 0.40. For $g \in \mathbf{G}^F$, let us write $g = t\pi(\tilde{g})$ with $t \in \mathbf{T}$ and $\tilde{g} \in \tilde{\mathbf{G}}$. Since g is F-fixed we have $t^{-1}\,{}^F t = \pi(\tilde{g}\,{}^F\tilde{g}^{-1}) \in \mathbf{T} \cap \pi(\tilde{\mathbf{G}}) = \pi(\tilde{\mathbf{T}})$, the last equality by the previous lemma. Since the kernel of π is contained in $\tilde{\mathbf{T}}$ this implies $\tilde{g}\,{}^F\tilde{g}^{-1} \in \tilde{\mathbf{T}}$. By Lang's theorem we can write $\tilde{g}\,{}^F\tilde{g}^{-1} = \tilde{t}_1^{-1}\,{}^F\tilde{t}_1$ with $\tilde{t}_1 \in \tilde{\mathbf{T}}$. Then $\tilde{t}_1\tilde{g}$ is in $\tilde{\mathbf{G}}^F$ and $t\pi(\tilde{t}_1^{-1}) = g\pi(\tilde{t}_1\tilde{g})^{-1}$ is in $\mathbf{T}^F$. Thus we have $g = t\pi(\tilde{t}_1^{-1})\pi(\tilde{t}_1\tilde{g}) \in \mathbf{T}^F\pi(\tilde{\mathbf{G}}^F)$, hence $\mathbf{G}^F = \mathbf{T}^F\pi(\tilde{\mathbf{G}}^F)$. Using the second assertion of the previous lemma we get $\mathbf{G}^F/\pi(\tilde{\mathbf{G}}^F) \simeq \mathbf{T}^F/\pi(\tilde{\mathbf{T}}^F)$. □

Now, since $X(\pi^*(\mathbf{T}^*))$ is generated by the roots, $Y(\tilde{\mathbf{T}})$ is generated by the coroots. Let θ be the character of $\mathbf{T}^F$ which corresponds to s. That s is orthogonal to the roots translates to the fact that θ vanishes on the images of the coroots, thus on $\pi(\tilde{\mathbf{T}}^F)$. Thus s defines some character $\hat{s}$ of the abelian group $\mathbf{G}^F/\pi(\tilde{\mathbf{G}}^F)$.

The character thus defined is independent of the torus used for its construction, since the characters obtained in various tori are geometrically conjugate, and

Proposition. *Geometric conjugacy is the identity on* $\mathbf{G}^F/\pi(\tilde{\mathbf{G}}^F)$.

Proof. Let $\mathbf{T}'$ be another F-stable torus and let n be such that there exists $x \in \mathbf{G}^{F^n}$ such that $\mathbf{T}' = {}^x\mathbf{T}$. We have to show that for $t \in \mathbf{T}^{F^n}$ the elements $N(t)$ and $N({}^x t)$ have same image in $\mathbf{G}^F/\pi(\tilde{\mathbf{G}}^F)$, where $N : \mathbf{G} \to \mathbf{G}$ is the map $y \mapsto y\,{}^F y \ldots {}^{F^{n-1}} y$.

Let $\phi_{\mathbf{G}} : \mathbf{T} \to \mathbf{G}$ be the map $t \mapsto N(t)^{-1}N({}^x t)$, and let $\phi_{\tilde{\mathbf{G}}}$ be the analogous map $\tilde{\mathbf{T}} \to \tilde{\mathbf{G}}$. We can lift $\phi_{\mathbf{G}}$ to $\pi \circ \phi$ where $\phi : \mathbf{T} \to \tilde{\mathbf{G}}$ is defined using that $\phi_{\mathbf{G}}$ (resp. $\phi_{\tilde{\mathbf{G}}}$) factors through $\mathbf{T}/Z(\mathbf{G})$ (resp. through $\tilde{\mathbf{T}}/Z(\tilde{\mathbf{G}})$) and that $\mathbf{T}/Z(\mathbf{G}) = \tilde{\mathbf{T}}/Z(\tilde{\mathbf{G}})$. This last fact since $\mathbf{T}/Z(\mathbf{G}) \simeq (\mathbf{T} \cap \mathbf{G}')/Z(\mathbf{G}')$ which is isomorphic by π to $(\tilde{\mathbf{T}} \cap \tilde{\mathbf{G}}')/Z(\tilde{\mathbf{G}}') \simeq \tilde{\mathbf{T}}/Z(\tilde{\mathbf{G}})$.

Take now $t \in \mathbf{T}^{F^n}$. Lift it to $\tilde{t} \in \tilde{\mathbf{G}}$, so that $\pi(\tilde{t}) = t$; then $\tilde{t}^{-1}\,{}^{F^n}\tilde{t} \in \ker\pi \subset Z(\tilde{\mathbf{G}})$. Using that for any y in an F-stable torus we have ${}^F N(y) = y^{-1}\,{}^{F^n}y N(y)$, we get ${}^F\phi_{\tilde{\mathbf{G}}}(\tilde{t}) = (\tilde{t}^{-1}\,{}^{F^n}\tilde{t})^{-1}\phi_{\tilde{\mathbf{G}}}(\tilde{t})^x(\tilde{t}^{-1}\,{}^{F^n}\tilde{t}) = \phi_{\tilde{\mathbf{G}}}(\tilde{t})$, the last equality since $\tilde{t}^{-1}\,{}^{F^n}\tilde{t}$ is central. Thus $\phi_{\mathbf{G}}(t) = \pi(\phi_{\tilde{\mathbf{G}}}(\tilde{t}))$ is in $\pi(\tilde{\mathbf{G}}^F)$. □

□

- page 118, replace the beginning of Remark 13.31 by the following

Remark. Actually it can be shown that $\mathbf{G}^F/\pi(\tilde{\mathbf{G}}^F)$ is the semi-simple quotient of the abelian quotient of $\mathbf{G}^F$ ...

- page 118, line 3: replace "is semi-simple" with "consists of semi-simple elements".

- page 127, line 21: replace "$|H^1(F, \mathbf{H})| = |\mathbf{H}^F|$" by "$|H^1(F, \mathbf{H})| = |(\mathbf{H}/\mathbf{H}^0)^F|$".

- page 129, line -10: The citation from Howlett is incorrect. B_l and C_l are exceptions over $\mathbb{F}_2$ for any l and G_2 over $\mathbb{F}_2$ is also an exception.

- page 130, definition 14.29: "For $z \in H^1(F, Z(\mathbf{G}))$" instead of "For $z \in Z(\mathbf{G})$".

- page 131, line 14: the unipotent radical of $\mathbf{P}$ should be denoted by $\mathbf{V}$ as stated lower; on line 15 u denotes an element of $\mathbf{V}$.

- page 132, last line of the proof of 14.32: "But then, by the choice of ψ_1, the result is clear." Unfortunately, it is not (clear that the restriction of ${}^n\psi_1$ is ψ_1). The proof of 14.32 shows that there exists z' such that ${}^*R_{\mathbf{L}}^{\mathbf{G}}(\Gamma_z^{\mathbf{G}}) = \Gamma_{z'}^{\mathbf{L}}$ but does not show that $z' = h_{\mathbf{L}}(z)$.

  Cédric Bonnafé has shown us the following way to fix the proof: since Harish-Chandra induction does not depend on the parabolic, we may as well choose for $\mathbf{P}$ the opposed parabolic; then the first place which changes in the proof page 131 is the computation of ${}^{n^{-1}}\mathbf{V} \cap \mathbf{U}$. We find this time that it equals ${}^{n^{-1}n_0}\mathbf{U} \cap \mathbf{U}$, where n_0 is a representative of the longest element of W. This contains no $\mathbf{U}_\alpha$ with $\alpha \in \Pi$ iff $w = 1$, that is $n \in \mathbf{T}^F$. This time it is indeed clear by definition that the restriction of Ψ_1 is Ψ_1.

  Comparison of the above proof with the one in the book shows also that when $N_{\mathbf{G}^F}(\mathbf{L})$ is generated by representatives of the elements $w_0^I w_0^J$ for $J \supset I$ (which happens when these elements normalize $\mathbf{L}$, which is for instance the case when $\mathbf{L}$ is "cuspidal", which means that for any proper Levi $\mathbf{M}$ of $\mathbf{L}$ the kernel of $h_{\mathbf{M}}$ is non-trivial), then $N_{\mathbf{G}^F}(\mathbf{L})$ acts trivially on the $\mathbf{T}^F$-orbits of regular characters of $\mathbf{U}^F \cap \mathbf{L}$.

- page 138, line 7:twice $\mathbf{T}^F$ instead of $\mathbf{T}$.

- page 142, add the following paragraph:

The results in this chapter which are specific to groups with non-connected centre come from our joint work with G. Lehrer [The characters of the group of rational points of a reductive group with non-connected centre, to appear in *Crelle's Journal*], who first exploited the role of $H^1(F, Z(\mathbf{G}))$ in his paper [On the characters of semisimple groups over finite fields, *Osaka Journal of math.* **15** (1978), 77–99].

- page 146, line 6: (j', i') instead of (i', j').

- page 149, line 13: $\mathrm{Id}_{\mathbf{G}}$ instead of $\mathrm{Id}_{\mathbf{G}^F}$.

- page 150, before 15.9 insert the following text:

Once we know the unipotent characters of $\mathbf{G}^F$, we can easily get all characters, using 13.30. Indeed we can take $(\mathbf{G}^*, F^*)$ to be $(\mathbf{G}, F)$; see examples above 13.11. Moreover the centralizer of a semi-simple element is a Levi subgroup by 2.6, and is isomorphic to a group of block-diagonal matrices. If s is rational semi-simple, by 4.3 the action of F on $C_{\mathbf{G}}(s)$ permutes blocks of equal size and the smallest power of F which fixes a block still acts on that block as a standard or unitary type Frobenius endomorphism (on a bigger field), except that in the unitary case an even power gives rise to the standard Frobenius endomorphism. Theorem 15.8 can be extended easily to such groups. Then, as by 13.25 $R^{\mathbf{G}}_{C_{\mathbf{G}}(s)}$ is an isometry from the series $\mathcal{E}(C_{\mathbf{G}}(s)^F,(s))$ to $\mathcal{E}(\mathbf{G}^F,(s))$, we get

Theorem. *The irreducible characters of the linear and unitary groups are (up to sign) the*

$$R_\chi(s) = |W_I|^{-1} \sum_{w\in W_I} \tilde{\chi}(ww_1) R^{\mathbf{G}}_{\mathbf{T}_{ww_1}}(s),$$

where $C_{\mathbf{G}}(s)$ is a Levi sugroup parametrized by the coset $W_I w_1$ as in 4.3. The character χ runs over w_1-stable irreducible characters of W_I and $\tilde{\chi}$ stands for an extension to $W_I.\langle w_1\rangle$ of χ.

- page 154, replace lines -6 to -1 by:

Since $\chi^+_{\omega_0}$ and $\chi^-_{\omega_0}$ are cuspidal, we have $D_{\mathbf{G}}(\chi^+_{\omega_0}) = -\chi^+_{\omega_0}$ and $D_{\mathbf{G}}(\chi^-_{\omega_0}) = -\chi^-_{\omega_0}$. We have $D_{\mathbf{G}}(\chi^+_{\alpha_0}) = R^{\mathbf{G}}_{\mathbf{T}}{}^*R^{\mathbf{G}}_{\mathbf{T}}\chi^+_{\alpha_0} - \chi^+_{\alpha_0}$; but ${}^*R^{\mathbf{G}}_{\mathbf{T}}\chi^+_{\alpha_0} = \alpha_0$ since ${}^*R^{\mathbf{G}}_{\mathbf{T}}R^{\mathbf{G}}_{\mathbf{T}}(\alpha_0)) = 2\alpha_0$ and $\langle R^{\mathbf{G}}_{\mathbf{T}}\alpha_0, \chi^+_{\alpha_0}\rangle_{\mathbf{G}^F} = 1$ so we get $D_{\mathbf{G}}(\chi^+_{\alpha_0}) = \chi^-_{\alpha_0}$ and similarly $D_{\mathbf{G}}(\chi^-_{\alpha_0}) = \chi^+_{\alpha_0}$; so (1) gives

$$\chi^+_{\alpha_0}(u_z) = \chi^-_{\alpha_0}(u_1) = -\chi^+_{\omega_0}(u_1) = -\chi^-_{\omega_0}(u_z) = -\sigma_1$$

and

$$\chi^+_{\alpha_0}(u_1) = \chi^-_{\alpha_0}(u_z) = -\chi^+_{\omega_0}(u_z) = -\chi^-_{\omega_0}(u_1) = -\sigma_z.$$