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A course on linear recurrent sequences
African Institute for Mathematical Sciences (AIMS)

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Tutorial 2

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• **1.** Exercise 4 of tutorial 1.
Consider an equilateral triangle having its vertices on a regular square grid with squares of side 1.
(a) Prove that the area of this triangle is a rational number.
(b) Let a be the length of the side of the triangle. Check that a^2 is an integer. Compute the area of the triangle in terms of a.
(c) Check that 3 does not divide a sum of two squares of relatively prime integers.
(d) Can you draw an equilateral triangle on the screen of a computer ?

• **2.** Questions of the students.
(a) *How can we find all the solution of the equation $X^2 - dY^2 = -1$, for a given d when a solution exist ?*
(b) *I would like to know a little bit more about the infinite descent of Fermat.*
(c) *What is the condition on an irrational number to have a periodic decomposition in continued fraction ?*
(d) *For a fixed integer d, I would like to know if the equation $X^2 - dY^2 = 1$ has always a non trivial solution ?*

• **3.** The triangle of sides $(3, 4, 5)$ is a rectangle triangle with hypotenus 5, the two sides of the right angle are consecutive integers, 3 and 4. Let $(c_n)_{n\geq 1}$ be the sequence, starting with $c_1 = 5$, of the integers which are the hypotenus of a right angle triangle where the two sides of the right angle are consecutive integers. Compute c_2. For $n \geq 3$, write c_n as a linear combination of c_{n-1} and c_{n-2}. Compute c_3 and c_4.

• **4.** Prove that every positive integer is the sum, uniquely, of one or more distinct Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers.