

Michel VAN DEN BERGH

(University of Hasselt and Free University of Brussels)

Counter examples in the theory of derived and triangulated categories



Jeudi 06

 février 2020

 17h00-18h00

 Université de Paris,

 Bâtiment Sophie Germain,

 salle 2015 (2e étage)

Derived categories were introduced by Grothendieck and Verdier as a language to state various duality theorems. They are obtained from categories of complexes by formally inverting «quasi-isomorphisms». Unfortunately derived categories do not admit many functorial constructions as they ignore higher homotopy information. In particular one has little control over the functors between them.

Nonetheless a celebrated theorem by Orlov, proved almost 25 years ago, provided some hope that, at least in algebraic geometry, derived categories might be rich enough after all. It states that any (exact) fully faithful functor between derived categories of coherent sheaves $D(X)$, $D(Y)$ on smooth projective varieties is a so-called «Fourier-Mukai functor». I.e. it is induced from a unique object living in $D(X \times Y)$.

Sadly, it appears that this is as far as it gets. We are now able to routinely construct counter examples against Orlov's result when the full faithfulness hypothesis is dropped. Ultimately the existence of such counter examples hinges on precisely controlling the (in)compatibility of functors with higher homotopy information. Recently we developed some machinery to conveniently handle this and this has also allowed us to construct new families of triangulated categories without «enhancement». Previously, essentially only a single example of this type, due to Muro, Schwede and Strickland, was known.

This is joint work with Amnon Neeman, Theo Raescheders and Alice Rizzardo.