

We generalize in  $\mathbf{C}^n$  Hilbert Lemniscate Theorem and we obtain several applications.

**Theorem.** *Let  $K$  be a polynomially convex compact subset of  $\mathbf{C}^n$  and let  $\mathcal{U}$  be an open neighbourhood of  $K$ .*

*Then there exists a proper polynomial mapping  $F = (p_1, \dots, p_n)$  of degree  $d$  such that*

$$K \subset \mathcal{P} \subset \mathcal{U}$$

*where the special polynomial polyhedron  $\mathcal{P}$  is a finite union of connected components of the open neighbourhood  $\{z \in \mathbf{C}^n : \sup_{1 \leq l \leq n} |p_l(z)| < 1\}$  of  $K$ .*

*In addition, the polynomial mapping  $F$  can be chosen such that:*

*(i) the homogeneous component  $H := (\hat{p}_1, \dots, \hat{p}_n)$  of degree  $d$  of  $F$ , (where  $\hat{p}_l$  for  $1 \leq l \leq n$  is the homogeneous component of degree  $d$  of the polynomial  $p_l$ ) is such that  $H^{-1}(\{O\})$  is reduced to  $\{O\}$ ,*

*(ii)  $F$  has a finite set  $Z$  of zeros in  $\mathbf{C}^n$  and each zero is of multiplicity one,*

*(iii) According to Bezout's Theorem,  $d^n = \text{card}(Z)$ . Here, we obtain that  $\frac{\text{card}(Z \cap \mathcal{P})}{d^n}$  is as near 1 as we want.*

A consequence of this result is an approximation of the pluricomplex Green function for the set  $K$  with pole at infinity, by maximal plurisubharmonic functions in  $\mathcal{L}^+$  with logarithmic poles in  $\mathcal{P}$ .

We obtain also a result about uniform polynomial approximation of holomorphic functions in a neighbourhood of  $K$ .

We study in details the particular case where  $K$  is balanced and it gives us the following result.

**Theorem.** *Let  $D = \{z \in \mathbf{C}^n : h(z) < 1\}$  be a bounded pseudoconvex balanced domain in  $\mathbf{C}^n$ . Let  $K$  be its closure. Then for any  $\epsilon > 0$ , there exists a proper homogeneous polynomial mapping  $F_\epsilon = (p_1, \dots, p_n)$  of degree  $d_\epsilon$  such that  $F_\epsilon^{-1}(\{O\})$  is reduced to  $\{O\}$  and*

$$K \subset \mathcal{P}_\epsilon \subset \{z \in \mathbf{C}^n : h(z) < 1 + \epsilon\},$$

*where  $\mathcal{P}_\epsilon$  is the domain  $\{z \in \mathbf{C}^n : \sup_{1 \leq l \leq n} |p_l(z)| < 1\}$ .*

*The family  $(\max_{1 \leq n} \frac{1}{d} \log |p_l|)_\epsilon$  approximates uniformly  $g_K = \log^+ h$  outside of  $K$ .*