

VARIÉTÉS DE CALABI-YAU GÉNÉRALISÉES

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ABSTRACT

Calabi-Yau manifolds play a special role in Kähler Geometry and their interest goes well beyond mathematics: in fact they represent a possible (micro)local model of the space of Quantum Theories.

In the spirit of investigating the relations between symplectic and holomorphic theories, we consider *Generalized Calabi-Yau Manifolds* i.e. $2n$ -dimensional (compact) differentiable manifolds M , equipped with a triple (κ, J, ϵ) where:

- (1) κ is a symplectic structure
- (2) J is a κ -calibrated, not necessarily integrable, complex structure
- (3) $\epsilon \in \wedge_J^{n,0}(M)$ such that
 - (a) $\epsilon \wedge \bar{\epsilon} = (-1)^{\frac{n(n+1)}{2}} i^n \frac{\kappa^n}{n!}$
 - (b) $\nabla \epsilon = 0$, (∇ being the exterior covariant differential operator of the Chern connection relative to (κ, J)).

We shall discuss several properties of Generalized Calabi-Yau Manifolds, including results on their Langrangian and Special Lagrangian submanifolds, their moduli spaces, the special features in the case $n = 3$.

We shall consider some applications to mirror symmetry, present various non trivial examples, propose a certain number of open problems.