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Abstracts of the talks

Positivity properties of bundles associated to holomorphic fibration

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Abstract : Let X be a complex manifold fibered over another complex manifold Y, and let L be a line bundle over X. We assume that X is Kahler and that the fibration has compact fibers. If L is semipositive and the fibration is smooth, then the direct image of $L + K_X$ is a vector bundle, who's curvature can be estimated in terms of the curvature of L. This implies a plurisubharmonicity property of an associated Bergman kernel, which in turn gives positivity properties of the relative canonical bundle of the fibration. We generalize these results on the relative canonical bundle to general surjective maps, that are not necessarily smooth fibrations, and to line bundles L that are only pseudoeffective. We also discuss variants of these results for multiples of K_X. (This is partly joint work with Mihai PAUN.)

The hypoelliptic Dirac operator

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Abstract : If X is a compact spin or spin-c manifold, the Dirac operator is an elliptic first order differential operator acting on smooth twisted spinors. All the operators coming from de Rham or holomorphic Hodge theory are Dirac operators.

The purpose of the lecture will be to explain the construction of a hypoelliptic Dirac operator on the total space $\mathcal{X}$ of the tangent bundle of X. It depends on a parameter $b > 0$. As $b \to 0$, the operator converges in the proper sense to the classical Dirac operator, and as $b \to +\infty$, it 'converges' to the geodesic flow on TX. In a earlier work, we had obtained a deformation of the classical Hodge-de Rham theory. As a special case, we obtain here a deformation of the Dolbeault-Hodge theory.

Applications to holomorphic Ray-Singer torsion will be outlined. The R genus of Gillet and Soulé reappers in this context.

Long-time asymptotics for the Camassa–Holm and NLS equations

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Abstract : I will describe the long-time asymptotic behavior of the solution $u(x,t)$ of the Cauchy problem for the Camassa–Holm (CH) equation

$$u_t - u_{txx} + 2\omega u_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}$$

on the line with fast decaying initial data $u_0(x)$, ω being a positive parameter.

I will also describe the long-time asymptotic behavior of the solution $q(x,t)$ of the initial-boundary-value problem for the focusing nonlinear Schrödinger equation $\mathrm{i}q_t + q_{xx} + 2|q|^2 q = 0$, on the first quarter plane $x > 0$, $t > 0$ for fast decaying initial data $q_0(x)$ and *time-periodic* boundary value $g_0(t)$. In both cases the solution $u(x,t)$ (resp. $q(x,t)$) behaves differently in various sectors of the (x,t)-half-plane (resp. quarter-plane).

The methods are inverse scattering transform in a matrix Riemann-Hilbert formulation and Deift and Zhou's nonlinear steepest descent method.

Work in collaboration with Dmitry Shepelsky (CH) and with Alexander Its and Vladimir Kotlyarov (NLS).

Estimates on Monge-Ampère operators derived from the Corti-Ein-Mustaţa inequality

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Abstract : We prove a new a priori estimate for the complex Monge-Ampère operator, according to which the inverse of the exponential of a plurisubharmonic function with isolated poles can be bounded in terms of its Monge-Ampère mass. The estimate is closely related to an inequality of local algebra proved a few years ago by Corti, Ein and Mustaţa. The proof rests on the approximation of general plurisubharmonic functions by Berman kernels and on general results concerning the semicontinuity of plurisubharmonic singularities. No direct analytic proof is known at present.

Symplectic topology of Stein manifolds

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Abstract : Symplectic geometry and topology is crucial for understanding Morse-theoretic problems for plurisubharmonic functions, homotopy classification of Stein structures on a given smooth manifold, and other problems of complex analysis. On the other hand, complex analytic methods allows one to define the relevant symplectic invariants.

Fillable CR-structures, gluing, and the index theorem

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Abstract : A problem of great current interest in symplectic topology concerns the ways in which a compact 3-dimensional contact manifold can be realized as the boundary of a symplectic, or Stein manifold. In this talk we consider the problem of Stein fillings from the perspective of analysis. The space of such fillings has a natural stratification defined by a spectral invariant, called the relative index, of the CR-structure on the boundary induced from the complex structure of the Stein filling. Using a sub-elliptic boundary condition for the Spinc-Dirac operator, we give a formula expressing this relative index in terms of the fundamental analytic and topological invariants of the filling manifold. This is a special case of a new, rather general, index formula, which includes the Atiyah-Singer formula as another special case.

On the Abel-Radon transform of locally residual currents

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Abstract : We study the Abel-Radon transform R with respect to a family of complete intersections of a fixed multidegree $(d_1, \ldots, d_n)$ in P^N. Let U be an open set on the parameter space T, and $U^* = \cup_{t \in U} H_t$, and α a locally residual current of bidegree $(N, p), p = N - n$. Then if $R(\alpha)$ extends holomorphically to a greater domain $\tilde{U} \subset T$, then α extends in a unique way as a locally residual current to $\tilde{U}^*$. We give for this a "trace lemma" of the type : Let $q : D \times \mathbb{C}^p \to D$, the standard projection, $D \subset C^n$, and β a proper locally residual current of bidegree (N, p) on $D \times \mathbb{C}^p$. If $u_{i_1 \ldots i_p} := q_*(\beta y_1^{i_1} \ldots y_p^{i_p})$ extend holomorphically to $\tilde{D}$, for a finite number of multi-indices $I = (i_1, \ldots, i_p)$, then β extend in a unique way as a proper locally residual current of bidegree (N, p) to $\tilde{D} \times \mathbb{C}^p$.

Riemann surface laminations

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Abstract : I will discuss joint work with Nessim Sibony on Riemann surface laminations with singularities.

The integral Cauchy formula on symmetric Stein manifolds

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Abstract : Many years ago I learnt from Gennadi Henkin that each time when you have a Cauchy formula you can add something substantial to the picture obtained by other tools. Not long time ago I found one more confirmation of this observation. It turns out that there is a Cauchy integral formula on complex semi-simple Lie groups and, more general, on symmetric Stein manifolds which gives interesting new facts in harmonic analysis on them.

Geometric structure arising from minimal rational curves

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Abstract : This is a survey of some of my joint works with Ngaiming Mok on minimal rational curves on Fano manifolds with second Betti number 1. Fano manifolds are projective algebraic manifolds with ample anti-canonical bundles, or equivalently, compact Kaehler manifolds with positive Ricci curvature. Most important example is the complex projective space. On each Fano manifold, we have the notion of minimal rational curves, which is a natural generalization of the lines on the complex projective space. Our main philosophy is that a Fano manifold of second Betti number 1 is determined by their minimal rational curves. We study in what sense this philosophy works from various view points.

Analysis on the worm domain

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Abstract : We discuss joint work with Marco Peloso on an asymptotic expansion for the Bergman kernel of a type of worm domain. We prove failure of Condition R, pathological behavior of the kernel on the boundary diagonal, and L^p estimates for the Bergman projection.

Optimal Metrics, 4-Manifolds, and Complex Singular Spaces

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Abstract : A Riemannian metric on a smooth compact 4-manifold is called OPTIMAL if it is "as flat as possible", in the sense of minimizing the L^2 norm of the curvature tensor. I will present some new constructions of such metrics via twistor correspondences and deformations of singular varieties. One consequence is a complete topological classification of those simply connected 4-manifolds which admit a special class of optimal metrics called scalar-flat anti-self-dual metrics. However, the same circle of ideas also shows that vast numbers of smooth compact 4-manifolds simply do not admit optimal metrics at all.

Local index theory and Bergman kernel

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Abstract : Let X be a positive line bundle on a compact complex manifold X. The Bergman kernel $P_p(x, x')$ is the smooth kernel of the orthogonal projection P_p, from the space of smooth sections of L^p ($p \in \mathbb{N}$) on $H^0(X, L^p)$ the space of holomorphic sections of L^p.

In this talk, we will explain how to use the analytic localization techniques in local index theory to establish the asymptotic expansion of $P_p(x, x')$ as $p \to \infty$. The simple principal is that the existence of the spectral gap of the operators implies the existence of the asymptotic expansion of the corresponding Bergman kernel no matter X is compact or not, or singular. In fact, we can also relax X to be a symplectic manifold. Moreover, the techniques in local index theory give us a general and algorithmic way to compute the coefficents of the expansion.

Probability measure on the space of Jordan curves

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Abstract : Representation of smooth Jordan curve on the complex plane by a smooth diffeomorphism of the circle through Ahlfors-Beurling conformal sewing. Identification of smooth Jordan curve to the double coset of the group of diffeomorphism by the group of Mobius transformation. Canonical Riemannian metric on the double coset. Brownian motion on the group of homemorphism of the circle. Resolution by continuity method of conformal sewing for Holderian Jordan curves. Probability measure on Holderian Jordan curve and its quasi-invariance. Kirillov complex Kahler structure on the "manifold" Jordan curves, and its corresponding canonical line bundle.

The $\bar{\partial}$-approach to approximate inverse scattering at fixed energy in three dimensions

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Abstract : We develop the $\bar{\partial}$-approach to inverse scattering at fixed energy in dimension $d \geq 3$ of [Beals,Coifman 1985] and [Henkin,Novikov 1987]. As a result we propose a stable method for nonlinear approximate finding a potential v from its scattering amplitude f at fixed energy $E > 0$ in dimension $d = 3$. In particular, in three dimensions we stably reconstruct n-times smooth potential v with sufficient decay at infinity, $n > 3$, from its scattering amplitude f at fixed energy up to $O(E^{-(n-3-\varepsilon)/2})$ in the uniform norm as $E \to +\infty$ for any fixed arbitrary small $\varepsilon > 0$ (that is with almost the same decay rate of the error for $E \to +\infty$ as in the linearized case near zero potential).

This talk is based, mainly, on [R.G.Novikov, International Mathematics Research Papers 2005 :6, 287-349 (2005)].

Homotopy formulas for the $\bar{\partial}$ operator on concave subvarieties of $\mathbb{C}P^n$ and Complex Radon transform

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Abstract : In the talk we will discuss recent results obtained jointly with G. Henkin on the application of homotopy formulas for $\bar{\partial}$-operator to the theory of the complex Radon transform and integral representation formulas for solutions of systems of differential equations.

For any, not necessarily reduced, complex analytic subvariety of $\mathbb{C}P^n$

$$V = \{z \in \mathbb{C}P^n : P_1(z) = \cdots = P_m(z) = 0\},$$

and a linearly concave domain $D \subset \mathbb{C}P^n$ we construct the integral operators $J,\ E,\ K$ on the space $\mathcal{R}^{(n,n-1)}(V \cap D)$ of $(n, n-1)$ currents with support on $V \cap D$ such that for any $\bar{\partial}$-closed current $\phi \in \mathcal{R}^{(n,n-1)}(V \cap D)$ the following equality holds

$$\phi = \bar{\partial}(J\phi) + E\phi + K\phi,$$

where $E\phi$ and $K\phi = G \circ R\phi$ are $\bar{\partial}$-closed, Coleff-Herrera-Lieberman residue currents with supports respectively on V and on $V \cap D$, and $R\phi$ is the Radon transform of ϕ. If $\dim V \geqslant 2$, then $J\phi \in \mathcal{R}^{(n,n-2)}(V \cap D)$, otherwise $J\phi = 0$. If V is a complete intersection, then J, E, and K can be defined by explicit expressions.

The formula above generalizes several previous formulas (Martineau, Gindikin, Henkin, Polyakov) and has the following corollaries :

- description of the kernel of the Radon transform as the space of residue cohomologies of $V \cap D$ extendible to residue cohomologies of V,
- description of the image of the Radon transform as the space of holomorphic solutions of the system of equations $P_1\left(\frac{\partial}{\partial\xi}\right) f = \cdots = P_m\left(\frac{\partial}{\partial\xi}\right) f = 0$, where $\xi \in D^* \subset (\mathbb{C}P^n)^*$,
- inversion formula $K\phi = G \circ R\phi$, which implies integral representation formulas for solutions of a system of homogeneous differential equations with constant coefficients in D^* going back to ..., Herglotz, Petrovsky, Fantappiè, Leray, ..., Berndtsson, Passare, Henkin.

A Difference - Differential Analogue of the Burgers Equation and Schumpeterian Dynamics

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Abstract : The talk contains a survey of results related to the following difference-differential equation :

$$(*) \qquad \frac{dF_n}{dt} = \phi(F_n)(F_{n-1} - F_n)$$

where, for every t, $\{F_n(t), n = 0, 1, 2, \ldots\}$ is a probability distribution function and ϕ is a positive function on $[0, 1]$.

The equation $(*)$ arises as a simplified description of economic development taking into account creation and propagation of new technologies in an industry or cross-country technology transfers. It may be considered as an analogue of the Burgers equation. If ϕ is decreasing then any solution of the Cauchy problem for $(*)$ approaches to a wave-train. For a non-monotonic case any solution approaches with time to a sum of diffusion curves and wave trains moving with different speeds. We discuss a number of modifications and unsolved problems including a multi-dimensional generalization of $(*)$ as well as applications to the economic growth and inter-country convergence theories.

Proper J-holomorphic disks in Stein domains of dimension 2

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Abstract : The problem of embedding complex disks or general Riemann surfaces into complex manifolds has been well-known for a long time. The interest to the case of almost complex manifolds has recently grown due to a strong link with symplectic geometry. We present the following result. Let (M,J) be an almost complex manifold of complex dimension 2 and let G be a smooth domain in M defined by a global strictly plurisubharmonic function. Then there exists a proper J-holomorphic disc passing through a given point in given direction.

On zeros of higher derivatives of meromorphic functions of finite order

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Abstract : I would like to discuss about a conjecture in Nevanlinna theory (called Gol'dberg conjecture), which roughly reads as follows : The number of zeros of higher derivatives of meromorphic functions in the complex plane is bounded from below by the number of its poles. In this talk, we consider meromorphic functions of finite order, and discuss the conjecture above.

Fourier Transform of "Simple" Functions

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Abstract : The rate of Fourier approximation of a given function is determined by its regularity. For functions with singularities, even very simple, like the Heaviside step function, the convergence of the Fourier series is slow, and their reconstruction from the truncated Fourier data involves systematic errors ("Gibbs effect").

It was recently discovered in the work of D. Donoho, E. Candes, V. Temlyakov, R. de Vore, and others that an accurate reconstruction from the sparse measurements data (in particular, from the truncated Fourier data) is possible not only for regular functions, but rather for "compressible" ones - those possessing a sparse representation in a certain basis.

In many important applications (like Image Processing) the linear representation of the data in a certain fixed basis may be not the most natural starting point. Instead, we can approximate the data with geometric models, explicitly incorporating nonlinear geometric elements like edges, ridges, etc. Accordingly, instead of "linear sparseness" we use another notion of "simplicity", based on the rate of the best approximation of a given function by semi-algebraic functions of a prescribed degree.

The main subject of the present talk is that such "simple" functions can be accurately reconstructed (by a non-linear inversion) from their truncated Fourier or Moment data.