Positivity properties of bundles associated to holomorphic fibration.

Bo Berndtsson

Chalmers University, Sweden

bob@math.chalmers.se

Abstract : Let X be a complex manifold fibered over another complex manifold Y, and let L be a line bundle over X. We assume that X is Kahler and that the fibration has compact fibers. If L is semipositive and the fibration is smooth, then the direct image of $L + K_X$ is a vector bundle, who's curvature can be estimated in terms of the curvature of L. This implies a plurisubharmonicity property of an associated Bergman kernel, which in turn gives positivity properties of the relative canonical bundle of the fibration. We generalize these results on the relative canonical bundle to general surjective maps, that are not necessarily smooth fibrations, and to line bundles L that are only pseudoeffective. We also discuss variants of these results for multiples of K_X. (This is partly joint work with Mihai PAUN.)