Local index theory and Bergman kernel

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Abstract : Let X be a positive line bundle on a compact complex manifold X. The Bergman kernel $P_p(x, x')$ is the smooth kernel of the orthogonal projection P_p , from the space of smooth sections of L^p $(p \in \mathbb{N})$ on $H^0(X, L^p)$ the space of holomorphic sections of L^p .

In this talk, we will explain how to use the analytic localization techniques in local index theory to establish the asymptotic expansion of $P_p(x, x')$ as $p \to \infty$. The simple principal is that the existence of the spectral gap of the operators implies the existence of the asymptotic expansion of the corresponding Bergman kernel no matter X is compact or not, or singular. In fact, we can also relax X to be a symplectic manifold. Moreover, the techniques in local index theory give us a general and algorithmic way to compute the coefficients of the expansion.