**Homotopy formulas for the $\bar{\partial}$ operator on concave subvarieties of $\mathbb{C}P^n$ and Complex Radon transform**

**Peter Polyakov**
Department of Mathematics
University of Wyoming, Dept. 3036
1000 East University Avenue, Laramie, WY 82071

**Abstract :**

In the talk we will discuss recent results obtained jointly with G. Henkin on the application of homotopy formulas for $\bar{\partial}$-operator to the theory of the complex Radon transform and integral representation formulas for solutions of systems of differential equations.

For any, not necessarily reduced, complex analytic subvariety of $\mathbb{C}P^n$

$$V = \{z \in \mathbb{C}P^n : P_1(z) = \cdots = P_m(z) = 0\},$$

and a linearly concave domain $D \subset \mathbb{C}P^n$ we construct the integral operators $J,\ E,\ K$ on the space $\mathcal{R}^{(n,n-1)}(V \cap D)$ of $(n, n-1)$ currents with support on $V \cap D$ such that for any $\bar{\partial}$-closed current $\phi \in \mathcal{R}^{(n,n-1)}(V \cap D)$ the following equality holds

$$\phi = \bar{\partial}(J\phi) + E\phi + K\phi,$$

where $E\phi$ and $K\phi = G \circ R\phi$ are $\bar{\partial}$-closed, Coleff-Herrera-Lieberman residue currents with supports respectively on $V$ and on $V \cap D$, and $R\phi$ is the Radon transform of $\phi$. If $\dim V \geqslant 2$, then $J\phi \in \mathcal{R}^{(n,n-2)}(V \cap D)$, otherwise $J\phi = 0$. If $V$ is a complete intersection, then $J$, $E$, and $K$ can be defined by explicit expressions.

The formula above generalizes several previous formulas (Martineau, Gindikin, Henkin, Polyakov) and has the following corollaries :

- description of the kernel of the Radon transform as the space of residue cohomologies of $V \cap D$ extendible to residue cohomologies of $V$,
- description of the image of the Radon transform as the space of holomorphic solutions of the system of equations $P_1\left(\frac{\partial}{\partial\xi}\right) f = \cdots = P_m\left(\frac{\partial}{\partial\xi}\right) f = 0$, where $\xi \in D^* \subset (\mathbb{C}P^n)^*$,
- inversion formula $K\phi = G \circ R\phi$, which implies integral representation formulas for solutions of a system of homogeneous differential equations with constant coefficients in $D^*$ going back to ..., Herglotz, Petrovsky, Fantappiè, Leray, ..., Berndtsson, Passare, Henkin.