KdV, mKdV and eigenfunctions of Schroedinger operators

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Abstract : We investigate the relation between the Korteweg - de Vries and modified Korteweg - de Vries equations (KdV and mKdV), and find a new algebro-analytic mechanism, similar to the Lax L-A pair, which involves a first-order operator Q instead of the third-order operator A. In our framework, eigenfunctions of the Schrödinger operator L, whose time-dependent potential solves the KdV equation, evolve according to a linear first-order partial differential equation, giving explicit control over their time evolution. As an application, we establish global existence and uniqueness for solutions of the initial value problem for mKdV in classes of smooth functions which can be unbounded at infinity. These classes may even include functions which tend to infinity with the space variable.

We also give a new proof of the invariance of the spectrum of the 1-dimensional Schrödinger operator under the KdV flow. It is, in particular, applicable to the potentials in the classes of possibly unbounded functions described above.

The talk will be based on a joint work of T.Kappeler, P.Perry, T.Topalov and the speaker, see http ://arxiv.org/pdf/math.AP/0601237