

Fourier Transform of "Simple" Functions**Y. Yomdin**Department of Mathematics
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The rate of Fourier approximation of a given function is determined by its regularity. For functions with singularities, even very simple, like the Heaviside step function, the convergence of the Fourier series is slow, and their reconstruction from the truncated Fourier data involves systematic errors ("Gibbs effect").

It was recently discovered in the work of D. Donoho, E. Candes, V. Temlyakov, R. de Vore, and others that an accurate reconstruction from the sparse measurements data (in particular, from the truncated Fourier data) is possible not only for regular functions, but rather for "compressible" ones - those possessing a sparse representation in a certain basis.

In many important applications (like Image Processing) the linear representation of the data in a certain fixed basis may be not the most natural starting point. Instead, we can approximate the data with geometric models, explicitly incorporating nonlinear geometric elements like edges, ridges, etc. Accordingly, instead of "linear sparseness" we use another notion of "simplicity", based on the rate of the best approximation of a given function by semi-algebraic functions of a prescribed degree.

The main subject of the present talk is that such "simple" functions can be accurately reconstructed (by a non-linear inversion) from their truncated Fourier or Moment data.