

The hypoelliptic Dirac operator by Jean-Michel Bismut

If X is a compact spin or spin-c manifold, the Dirac operator is an elliptic first order differential operator acting on smooth twisted spinors. All the operators coming from de Rham or holomorphic Hodge theory are Dirac operators.

The purpose of the lecture will be to explain the construction of a hypoelliptic Dirac operator on the total space \mathcal{X} of the tangent bundle of X . It depends on a parameter $b > 0$. As $b \rightarrow 0$, the operator converges in the proper sense to the classical Dirac operator on X , and as $b \rightarrow +\infty$, it ‘converges’ to the geodesic flow on TX . In an earlier work, we had obtained a deformation of the classical Hodge-de Rham theory. As a special case, we obtain here a deformation of the Dolbeault-Hodge theory.

Applications to holomorphic Ray-Singer torsion will be outlined. The R genus of Gillet and Soulé reappears in this context.