Séminaire de théorie des nombres

Le 30 mars 2009 à 14h

Variants of classical Jacobi formula in two variables with application to new AGM theorems

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Résumé : The classical Jacobi formula for the elliptic integrals (Gesammelte Werke I, p.235) shows a relation between the Jacobi theta constant and periods of the ellptic curve $E(\lambda) : w^2 = z(z-1)(z-\lambda)$. In this talk we show a variant of this formula for several variables case.

In other words, the Jacobi formula shows that the modular form $\vartheta_{00}^4(\tau)$ with respect to the principal congruence subgroup $\Gamma(2)$ of $PSL(2,\mathbb{Z})$ has an expression by the Gauss hypergeometric function $F(\frac{1}{2},\frac{1}{2},1;1-\lambda)$ of the algebraic parameter λ via the inverse of the period map for the family of elliptic curves $E(\lambda)$. Our result is a two dimensional exact analogy of this context.

As an application we expose a new proof of the extended Gauss AGM formula with some numerical demonstration.