

Séminaire de théorie des nombres

Le 23 juin 2014 à 14h (Jussieu)

Arithmetic analogue of some concepts in Riemannian geometry

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Résumé : Starting with a symmetric/antisymmetric matrix with integer coefficients (which we view as an analogue of a metric/form on a bundle over $\text{Spec}(\mathbf{Z})$) we introduce arithmetic analogues of connections and curvature (in which usual derivatives acting on functions are replaced by Fermat quotient operators acting on integer numbers). The Christoffel symbols are then matrix analogues of Legendre symbols. We prove that the curvature of the connection attached to the matrix defining the split SO_n or Sp_n does not vanish for n at least 4. We also show that the curvature for these matrices vanishes "to order 3" for all n . Morally, $\text{Spec}(\mathbf{Z})$ is "curved" but only "mildly" curved. This and related results could be viewed as first steps in a program of developing an "arithmetic analogue of differential geometry".