

# Séminaire de théorie des nombres

Le 15 décembre 2014 à 14h (PRG)

## The proportion of failures of the Hasse norm principle

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**Résumé :** Let  $K$  be a number field and let  $J_K$  be its group of ideles. If a nonzero rational number is a norm from  $K^*$ , then it is a norm from  $J_K$ . We say that the Hasse norm principle holds for  $K/\mathbf{Q}$  if the converse holds, i.e. if every rational number which is a norm from  $J_K$  is in fact a norm from  $K^*$ .

This talk is about the proportion of rational numbers which are counterexamples to the Hasse norm principle for  $K/\mathbf{Q}$ . Using work of Odoni, we give asymptotic formulae for the counting functions for rational numbers that are norms from  $J_K$  and for rational numbers that are norms from  $K^*$ . We calculate the proportion of rational numbers that are norms from  $J_K$  which fail to be norms from  $K^*$ . We show that this proportion is  $1 - 1/n$ , where  $n$  is the (finite) index of  $N(K^*)$  in  $N(J_K) \cap \mathbf{Q}^*$ .

This is joint work with Tim Browning.