Séminaire de théorie des nombres

Le 26 février 2018 à 14h (Jussieu)

The higher Chow groups with modulus and cube invariance

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Résumé : The motivic cohomology of a variety X over a field is defined (at least) in two ways : the Hom-groups of Voevodsky's category of motives **DM**, or the homology groups of a cycle complex introduced by Bloch. Voevodsky proved a comparison isomorphism between them for any smooth X.

Recently, Kahn-Saito-Yamazaki extended Voevodsky's category **DM** to the category of motives with modulus **MDM**. The constructions of the categories are similar, but there is one essential difference : $\mathbb{A}^1$-homotopy invariance is replaced by $(\mathbb{P}^1, \infty)$-invariance (or "cube invariance"). The category **MDM** is large enough to contain all commutative algebraic groups over a field (more generally, reciprocity sheaves).

On the other hand, Binda-Saito introduced the higher Chow groups with modulus $\mathrm{CH}^r(X, D, *)$, as a non-$\mathbb{A}^1$-homotopical generalization of Bloch's higher Chow groups. They describe some non-$\mathbb{A}^1$-homotopy invariant objects (e.g. the big de Rham complexes of a field, the étale fundamental groups).

In this talk, I will formulate and state the cube invariance for the higher Chow groups with modulus, which may give us a hint to find a right generalization of Voevodsky's comparison theorem. If time permits, I will explain some results on the "gap" between cube invariance and $\mathbb{A}^1$-homotopy invariance.