## Séminaire de théorie des nombres

Le 28 mars 2022 à 14h (Jussieu)

## A density conjecture about unit fractions

## Exposé de Thomas Bloom (University of Oxford)

**Résumé :** In 2001 Croot resolved an old conjecture of Erdos and Graham, proving that in any finite colouring of the positive integers there is a (non-trivial) monochromatic solution to  $\frac{1}{n_1} + \cdots + \frac{1}{n_k} = 1$  with all  $n_i$  distinct. A natural generalisation, also conjectured by Erdos and Graham, is that in fact any set of positive density contains such a solution. We will discuss the proof of this conjecture, which extends Croot's method, and uses Fourier analysis coupled with elementary number theoretic and combinatorial arguments. We will also review several still open conjectures concerning unit fractions.