Formes modulaires, représentations galoisiennes et équations diophantiennes

Une rencontre en l'honneur d'Alain Kraus

Lundi 21 novembre 2022

Les exposés auront lieu en **salle 15–16 413** de l'Institut de Mathématiques de Jussieu – Paris Rive Gauche (site de Jussieu).

- **9h30–10h30 : Samir Siksek (Warwick)**

  The method of Kraus for differences of squares and perfect powers

  Résumé : Diophantine equations involving the difference between a square and an arbitrary perfect power have been studied by elementary means since 1850, and using Baker's theory of linear forms in logarithms since 1960s. We illustrate how the method of Kraus can be made to work hand-in-hand with the elementary and Baker techniques to solve some difficult instances of these problems.

- **11h–12h : Baptiste Peaucelle (Lyon)**

  Small images computations for modular Galois representations

  Résumé : Given a modular form f and a prime ideal λ in the coefficient field of f, one can attach a residual Galois representation of dimension 2 with values in the residue field of λ. A theorem of Ribet states that this representation has small image for a finite number of prime ideals λ. In my talk, I will explain how one can bound and compute these exceptional cases by checking congruences between modular forms.

- **14h–15h : Ekin Özman (Istanbul)**

  Modular Curves and Asymptotic Solutions to Fermat-type Equations

  Résumé : Understanding solutions of Diophantine equations over a number field is one of the main problems of number theory. By the help of the modular techniques used in the proof of Fermat's last theorem by Wiles and its generalizations, it is possible to solve other Diophantine equations too. Understanding quadratic points on the classical modular curve also plays a role in this approach. It is also possible to study the solutions of Fermat type equations over number fields asymptotically. In this talk, I will mention some recent results about these notions for the classical Fermat equation as well as some other Diophantine equations.

- **15h30–16h30 : Nuno Freitas (Madrid)**

  A first approach to the Darmon program for Fermat-type equations

  Résumé : In 2000, Darmon described a remarkable program to study the Generalized Fermat equation $Ax^r + By^q = Cz^p$ using modularity of abelian varieties of GL_2-type over totally real fields. However, his program relies on hard open conjectures, which has made it difficult to apply in practice, and so far the only successes were in cases where the Frey varieties are elliptic curves.

  In this talk, we will first discuss the modular method and its limitations; then we will discuss some of Darmon's ideas to circumvent these limitations and finally, we will sketch how combining ideas from Darmon's program with the classic Frey curve approach allows for a complete resolution of the generalized Fermat equation $x^7 + y^7 = 3z^n$ for all integers $n \geq 2$. An essential piece in this approach is a Frey hyperelliptic curve due to Alain Kraus.

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