Séminaire de théorie des nombres

Le 07 avril 2025 à 14h (Jussieu)

Bounding tangencies between sections of elliptic surfaces

Exposé de Douglas Ulmer
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Résumé : We take a section P of infinite order on an elliptic surface and consider points where some multiple nP is tangent to the zero section. (These are "unlikely intersections" and our consideration of them is motivated by a question in geography of surfaces. It is also analogous to the question of whether elements of an elliptic divisibility sequence are square-free.) In characteristic zero, we show finiteness and give a sharp upper bound, relying heavily on a canonical parallel transport in a family of elliptic curves (the "Betti foliation") and a certain real-analytic one-form. Although the finiteness statement looks completely reasonable in characteristic p, it's not clear what would replace the (non-algebraic) 1-form. Time permitting, I will explain how ongoing work with Felipe Voloch connects tangencies to the p-descent map and allows us to bound them in characteristic p as well. Joint work with G. Urzua and F. Voloch.