Séminaire de théorie des nombres

Le 23 juin 2025 à 14h (PRG)

The kernel of the adjoint exponential in Anderson t-modules

Exposé de Giacomo Hermes Ferraro (Heidelberg University)

Résumé : Given an algebraically closed complete valued field K over $\mathbb{F}_q$, an Anderson t-module of dimension d is given by the topological $\mathbb{F}_q$-vector space K^d, endowed with an $\mathbb{F}_q$-linear action $\phi_t = \sum_{i\geq 0} T_i\tau^i \in M_{d\times d}(K)[\tau]$, where $\tau : K^d \to K^d$ sends $(v_1, \dots, v_d)$ to $(v_1^q, \dots, v_d^q)$. In analogy with complex abelian varieties, there is an analytic map $\exp = \sum_{i\geq 0} E_i\tau^i : K^d \to K^d$—which is not necessarily surjective—such that $\phi_t \exp = \exp T_0$.

The adjoint exponential, defined as the series $\exp^* := \sum_{i\geq 0} \tau^{-i}E_i^T$, determines a (non-analytic) continuous map $K^d \to K^d$. Using the factorization properties of $K[\![x]\!]$, Poonen proved that there is a perfect duality of topological $\mathbb{F}_q$-vector spaces $\ker(\exp) \times \ker(\exp^*) \to \mathbb{F}_q$ under the condition $d = 1$.

In this talk, I explain that for an arbitrary *abelian* Anderson t-module, we have a collection of perfect pairings $\ker(\phi_{t^n}) \times \ker(\phi^*_{t^n}) \to \mathbb{F}_q$, and that we can use them to obtain a canonical generating series $(F_\phi)_c \in M_{d\times d}(K)[\![\tau^{-1}, \tau]\!]$ for all $c \in \mathbb{F}_q(\!(t^{-1})\!)/\mathbb{F}_q(t)$. The study of the properties of F_ϕ allows us to prove that, if exp is surjective, $\ker(\exp^*)$ is compact and isomorphic to the Pontryagin dual of $\ker(\exp)$. Moreover, we deduce an alternative explicit description of the Hartl–Juschka pairing, obtained by Gazda and Maurischat in a recent preprint.