

Séminaire de théorie des nombres

Le 24 novembre 2025 à 14h (Jussieu)

Proétale motives and rigidity

Exposé de Swann Tubach
(IMJ-PRG)

Résumé : A very efficient way of constructing the ℓ -adic realisation functor on categories of motives is proving a rigidity theorem : according to Suslin-Voevodsky, Ayoub, Cisinski-Dégliise and Bachmann (in increasing generality), given a prime number ℓ , the ℓ -adic realisation functor is just the ℓ -completion functor on étale motivic sheaves. This construction is sufficient in most situations, but has some flaws, as for example it is not compatible with tensoring with the rational numbers, making the construction of a $\mathbb{Q}_\ell$ -adic realisation functor unnatural. We will define categories of proétale motives, give their basic properties, and explain how to prove a similar rigidity statement for a suitably solidified version of the categories, in a way that fixes the above flaw. A consequence of this rigidity theorem is that ℓ -adic solid sheaves à la Fargues-Scholze, whose definition can be modified to work over schemes, afford the 6 operations, and a realisation functor from motives, as they agree with solid ℓ -adic proétale motives. This is joint work with Raphaël Ruimy and Sebastian Wolf.