

Séminaire de théorie des nombres

Le 18 février 2019 à 14h (PRG)

Shimura curves and the *abc* conjecture

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Résumé : Elliptic curves over the rationals admit maps from various Shimura curves, and the comparison ratio of the degrees of these maps recovers important information on *abc*-triples. On the other hand, this ratio can be controlled by the Arakelov height of CM points. This requires a number of tools : zero-density estimates for L-functions, integral models for various objects, Galois representations, and some complex-analytic estimates. The final outcome is an unconditional estimate for the product of p -adic valuations of *abc*-triples, which lies beyond the reach of existing methods in the context of the *abc* conjecture such as linear forms in logarithms. Our methods also yield other results. For instance, for totally real fields F of bounded degree, we prove that the Faltings height of modular elliptic curves E over F is bounded linearly on $\log(\text{modular degree of } E) + \log(\text{Disc. of } F)$. The logarithmic dependence of the discriminant of F can be seen as evidence towards Vojta's conjecture on algebraic points of bounded degree.