Proceedings of the International Congress of Mathematicians
Helsinki, 1978

Integral Methods and Zeros of Holomorphic Functions

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Introduction. The entire functions are so closely related with the theory of systems of partial differential equations with constant coefficients and of convolution equations (cf. [**1**], [27], [29]), that about 1950 a deep study of their properties seemed necessary to many mathematicians. Especially, the connection between the growth of entire functions and the growth of the sets of zeros appeared of great interest. For one variable, it is easy to deal with the Jensen formula and with the Weierstrass canonical products, in a very classical way. For several variables there is no trivial extension of the Weierstrass products but we are able to build entire functions with a given set of zeros and whose growth is connected with the growth of zeros. Working on these problems, the mathematicians were led to use more systematically tools of complex analysis as positive closed currents, plurisubharmonic functions, L^2 estimates or integral methods for the $\bar\partial$ operator.

1. The current of integration and the Lelong—Poincaré equation. Let X be an analytic subset of dimension p in a complex manifold of dimension n. In 1953, P. Lelong [**23**] has proved that it is possible to define a current $[X]$ using integration over the regular part $\operatorname{Reg} X$ of X:

(1) $$\langle [X], \varphi\rangle = \int_{\operatorname{Reg} X} \varphi,$$

where φ is a form of total degree $2p$. Besides $[X]$ is closed, of bidegree $(n-p, n-p)$ and positive; a current θ of bidegree $(n-p, n-p)$ is said to be positive if:

$$i^p \theta \wedge \alpha_1 \wedge \bar\alpha_1 \wedge \ldots \wedge \alpha_p \wedge \bar\alpha_p \geqslant 0,$$

for all $(1, 0)$ forms α_j (the orientation of the manifold is defined by the (n, n) form $i^n dz_1 \wedge d\bar z_1 \wedge \ldots \wedge dz_n \wedge d\bar z_n$).

Let in C^n be: $\alpha = i\partial\bar{\partial}\log|z|^2/2$, $\beta = i\partial\bar{\partial}|z|^2/2$, let σ and ν be the positive measures associated to θ by:

(2) $$\sigma = \frac{1}{p!}\theta\wedge\beta^p, \quad \nu = \pi^{-p}\theta\wedge\alpha^p.$$

Let $\sigma(r)$ be the measure σ supported by the closed euclidean ball of radius r. If $\theta = [X]$, σ is the $2p$-dimensional area of X. The significant growth of θ (resp. $[X]$) is measured by the projective indicator: $\nu(r) = \pi^{-p} p!\, r^{-2p}\sigma(r)$.

The growth of an entire application F of C^n in C^k will be described by:

$$M_F(r) = \sup_{|z|\leq r}\|F(z)\|.$$

When X is a hypersurface which is defined by only one equation F, then X and F are connected by the Lelong–Poincaré equation of currents:

(3) $$\frac{i}{\pi}\partial\bar{\partial}\log|F| = \sum_j n_j[X_j] = \theta$$

(where n_j is the multiplicity of F on the irreducible branch X_j of X).

With the Poisson–Jensen formula, it is possible exactly as in the case of one variable, to obtain a bound for $\nu(r)$:

$$\nu(r) \leqslant C(\varepsilon, n)\log M_F(r+\varepsilon r),$$

for all $\varepsilon > 0$ (and supposing $|F(0)| = 1$ for the simplicity). Conversely, if V is a solution of the equation:

(4) $$i\partial\bar{\partial}V/\pi = \theta,$$

then P. Lelong observed in [24] that necessarily $V = \log|F|$, for some entire function F such that $F^{-1}\{0\} = X$.

When θ is of finite order (i.e. $\limsup_{r\to+\infty} r^{-\varrho}\nu(r) < +\infty$ for some $\varrho \geqslant 0$), P. Lelong [24] has built in 1953 an explicit canonical potential V, using a modification of the kernel $-|z-x|^{-2n+2}$ by harmonic terms, which is a solution of (4). This potential exactly generalizes the canonical Weierstrass product. The difficulty is to prove that V, which is already a solution of $4\Delta V = \sigma$, is in fact a solution of (4).

Therefore, using V we can define a given X by an entire function of the same order as X. W. Stoll, using H. Kneser's work [21], reaches the same result in [39], but does not obtain a globally convergent representation for $\log|F|$.

In 1970, in [33], [34], we dealed with the general case, without restriction about X. We resolved the equation (4), using a regularization of θ, the classical Cartan–Poincaré homotopy formula for $d = \partial + \bar{\partial}$ and the Hörmander's L^2 estimates for $\bar{\partial}$. We dealt also with the case where F verifies the weaker condition $F^{-1}\{0\} \supset X$, but not necessarily $F^{-1}\{0\} = X$.

E. Bombieri in [2] (1970) has given a beautiful application of Lelong's results to the arithmetical properties of entire functions. This work will be pursued by M. Waldschmidt [43].

2. The case of analytic sets of codimension > 1. When the analytic set X is a complete intersection, defined as the set of zeros of a holomorphic map $F=(F_1, F_2, \ldots, F_{n-p})$ of C^n (or of a manifold) in C^{n-p}, P. A. Griffiths and J. R. King **[10]** proved in 1973 the following "Poincaré–Martinelli" equation of currents:

$$(2\pi)^{-k} i\,\partial\bar{\partial}\,[\log \|F\|^2 (i\partial\bar{\partial} \log \|F\|^2)^{k-1}] = [X], \tag{5}$$

where $k=n-p$, where the form in the brackets has locally summable coefficients, and where each irreducible branch of X is counted with the appropriate multiplicity of F (this multiplicity is defined for example in [7]). But the relation (5) is less useful than the Lelong–Poincaré equation, as well to obtain a bound of $\nu(r)$ when a bound of $M_F(r)$ is given, as well to try to build the functions F_j when X is given.

A counterexample of M. Cornalba and B. Shiffman, in 1972 **[5]**, shows that it is not possible in general to obtain a bound of $\nu(r)$ when a bound of $M_F(r)$ is given. Precisely, they proved:

THEOREM 1. *Let* $s\colon \mathbf{R}_+ \to \mathbf{R}_+$ *be an increasing function. There exists a holomorphic map* F *of* $\mathbf{C}^2$ *in* $\mathbf{C}^2$ *such that* $X=F^{-1}\{0\}$ *has dimension* 0, *such that* $M_F(r)$ *is of order zero, but such that* $\nu(r)$ *grows faster than* $s(r)$, *that is:*

$$\lim_{r\to+\infty} \frac{\nu(r)}{s(r)} = +\infty,$$

$$\limsup_{r\to+\infty} r^{-\varrho} \operatorname{Log} M_F(r) = 0, \quad \textit{for all} \quad \varrho > 0.$$

This means that the Bezout theorem over the algebraic curve has no transcendental equivalent. Nevertheless, different works have been done by W. Stoll **[41]**, P. A. Griffiths **[11]** and L. Gruman **[13]** in order to obtain weaker forms of bounds of "transcendental Bezout theorem" type.

It is therefore surprising that the inverse problem, which seems at first more difficult, received a positive general answer, given by the author in 1972 in **[35]**. We have:

THEOREM 2. *Let* X *be an analytic subset of* $\mathbf{C}^n$, *such that* $0 \notin X$, *let* ε *and* $\delta \in \mathbf{R}^+$. *Then there exist* $n+1$ *entire functions* $F=(F_1, F_2, \ldots, F_{n+1})$ *such that* $F^{-1}\{0\}=X$ *and such that for all* $r>0$, F *verifies one of the following bounds:*

1. $\log M_F(r) \le C(\varepsilon)\, \nu(r+\varepsilon r) \cdot \log^2 r$,
2. $\log M_F(r) \le C(\varepsilon, \delta)\, r^\delta \int_0^r t^{-\delta-1} \nu(t+\varepsilon t)\, dt$,

where $C(\varepsilon)$ *and* $C(\varepsilon, \delta)$ *are independent from* r.

Particularly, if $\nu(r) \le C_1 r^\varrho$, then we are able to find F such that $\log M_F(r) \le C_2 r^\varrho$, for some constant C_1 and C_2 (choose $\delta < \varrho$). We obtained in **[35]** different technical bounds which we do not reproduce for simplicity and which especially give the following result of W. Stoll [40] and E. Bishop [1] (1966) about algebraic subset of C^n.

THEOREM 3. *X is algebraic if and only if $r \mapsto \nu(r)$ is a bounded function.*

We shall now briefly talk about the methods of the proof. We do not use (5) which supposes that X is a complete intersection. We begin to build a plurisubharmonic function U which behaves intuitively as $\log \|F\|$, where F is the requested map. At first, we locally build U as an explicit negative potential, by integration of the kernel $-|z-x|^{-2p}$ over X. Then using a partition of unity, we obtain a global $\leqslant 0$ potential U_0 which is nearly plurisubharmonic (i.e. modulo C^∞ strictly plurisubharmonic functions). We estimate the lack of plurisubharmonicity of U_0 and we add to U_0 a strictly plurisubharmonic function with controlled growth to get U. When X has a low growth, such that $\int_1^{+\infty} t^{-2}\nu(t)\,dt < +\infty$ and when $0 \notin X$, we can take very simply (and globally):

$$U(z) = \int_{C^n} [-|z-x|^{-2p} + |x|^{-2p}]\beta^p \wedge \theta(x), \tag{6}$$

where $\theta = [X]$. The difficulty is to prove that U is plurisubharmonic. To be successful we represent U as a direct image of a current on $C^n \times C^n$. Let π_1 and π_2 be the projections of $C^n \times C^n$ over C^n, τ the diagonal map $(x, z) \to z-x$, and $K = -|x|^{-2p}\beta^p$. We prove that choosing a convenient C^∞ function $\chi \geqslant 0$, which is equal to 1 on a neighbourhood of the diagonal of $C^n \times C^n$, we have:

$$U = \pi_{2*}(\chi\tau^* K \wedge \pi_1^*\theta) + \text{technical terms},$$
$$i\partial\bar{\partial} U = p\pi_{2*}(\chi\tau^*\alpha^{p+1} \wedge \pi_1^*\theta) + \text{technical terms},$$

such that the positivity of $i\partial\bar{\partial}U$ is a consequence of the positivity of α and θ and of the fact that positivity is invariant through the direct and inverse image (modulo precise estimates for the technical terms, cf. [34] and [35]). Now we build the functions $F_1, F_2, \ldots, F_{n+1}$ using a theorem of Hörmander–Bombieri ([2], [1]) which gives (because of the plurisubharmonicity of U) the existence of a non trivial entire function f such that

$$\int_{C^n} |f|^2 \exp(-lU)(1+|z|^2)^{-n-2}\,d\lambda(z) < +\infty, \tag{7}$$

where $l > 0$ and where $d\lambda$ is the Lebesgue measure.

U were built such that for some $l > 0$, $\exp(-lU)$ is not locally summable in all points of X. Therefore the estimates (7) imply that f is null over X. Besides the estimates (7) give bounds for the growth of f. We repeat a classical argument of H. Grauert in order to obtain $n+1$ functions satisfying (7) such that $F^{-1}\{0\} = X$.

The results of Theorem 2 seem the best from the point of view of the comparison of growth of F and X. There is still an open problem, that is to reduce the number $n+1$ of functions defining X in theorem 2 without loss of growth. According to O. Forster and K. J. Ramspott [9] n functions are always sufficient to define X (without controlled growth). When X is a submanifold of C^n, topological conditions which were shown in **[10]** gives the insurance that X is a complete intersection (for instance in codimension 2, the necessary and sufficient condition is the vanishing of the first Chern class of X). New ideas are requested to obtain the equivalent of Theorem 2 in these cases.

The construction of the potential U associated to X in the proof of the Theorem 2 is much generally valid for an arbitrary positive, closed current θ. Therefore to each such $(n-p,\ n-p)$ current is associated a plurisubharmonic function U and a positive (1, 1) current $i\partial\bar{\partial}U$, whose properties are intimately connected with the properties of θ. The density or Lelongs number of θ at z is defined by:

$$\nu(z) = \lim_{r\to 0} \pi^{-p} p!\, r^{-2p} \int_{B(z,r)} \beta^p \wedge \theta$$

($B(z, r)$ is the euclidean closed ball of radius r centered at z). $\nu(z)=0$ if θ is C^0 in a neighbourhood of z. If $\theta=[X]$, $\nu(z)$ is an integer at each point $z\in X$ and $\nu(z)=1$ if $z\in \operatorname{Reg} X$. Conversely, the following result of Y. T. Siu **[32]** (1974), whose particular case is solved by Bombieri **[2]**, King **[15]**, Harvey **[14]**, Skoda **[38]** says how much of a given θ is an analytic set.

THEOREM 4. *Let θ be a positive, closed, (k, k) current on a complex manifold Ω. For all $c>0$, the set $E_c=\{z\in\Omega | \nu(z)\geqslant c\}$ is an analytic subset of Ω (of smaller dimension than $n-k$).*

The proof heavily depends on the Hörmander–Bombieri result (7).

Recently, P. Lelong **[25]** has proved that the (1, 1) current $i\partial\bar{\partial}U$, locally associated to θ as in (6), has the same density as θ in all points. Therefore, it is sufficient to prove Theorem 4 for a (1, 1) current. In this last case, the proof of Y. T. Siu in **[32]** is particularly elegant.

The result of Theorem 2 has easy extensions to an open pseudoconvex subset of C^n and to Stein manifolds (cf. H. Skoda **[35**, Proposition 9.1**]**) but the precision of the bounds is limited by the L^2 estimates and is not always the best. Therefore new methods were necessary.

3. Fine results for strictly pseudoconvex open sets in C^n. Let Ω be a bounded, strictly pseudoconvex, open set in C^n, of class C^2, that is: $\Omega=\{z | \varrho(z)<0\}$, where ϱ is a real function, defined, of class C^2, strictly plurisubharmonic in a neighbourhood of $\bar{\Omega}$ and verifying $d\varrho\neq 0$ on $\partial\Omega$. Let Ω_ε be the set $\{z | \varrho(z)<-\varepsilon\}$.

The Nevanlinna class $N(\Omega)$ (resp. the space $H^p(\Omega)$, $0<p<+\infty$) is the set of holomorphic functions f on Ω such that:

$$\limsup_{\substack{\varepsilon\to 0\\ \varepsilon>0}} \int_{\partial\Omega_\varepsilon} \log^+ |f|\, dS_\varepsilon < +\infty, \tag{8}$$

resp.

$$\limsup_{\substack{\varepsilon\to 0\\ \varepsilon>0}} \int_{\partial\Omega_\varepsilon} |f|^p\, dS_\varepsilon < +\infty,$$

where dS_ε is the euclidean area of $\partial\Omega_\varepsilon$. We trivially have

$$H^\infty(\Omega)\subset H^p(\Omega)\subset N(\Omega),$$

for all p, where $H^\infty(\Omega)$ is the space of bounded functions. An hypersurface X verifies the Blaschke condition if by definition:

$$\int_X |\varrho(z)|\, d\sigma(z) < +\infty, \tag{9}$$

where $d\sigma$ is the area element on X. For one variable, it is classical that the Blaschke condition (9) characterizes the set of zeros of functions of $N(\Omega)$ and of all $H^p(\Omega)$ $0<p\leq+\infty$. For several variables, it is easy to prove using the Poisson–Jensen formula (cf. [4]) that the zero set of $f\in N(\Omega)$ verifies (9). But W. Rudin [31] has proved [11] that the characterization of zeros of functions in $H^p(\Omega)$ for the euclidean ball must necessarily depend on p, so that such a characterization is probably much more complicated as in the case $n=1$. Nevertheless, in 1975 G. M. Henkin [17] and the author [37], [38] have independently but by very similar methods proved the following:

THEOREM 5. *Let Ω be a strictly pseudoconvex open set such that $H^2(\Omega, \mathbf{Z})=0$ and let X be an hypersurface of Ω verifying the Blaschke condition, then X is the zero set of some $f\in N(\Omega)$.*

More generally, we solved the Lelong–Poincaré equation $i\partial\bar\partial V=\theta$, where θ is a given (1, 1) positive, closed, current verifying:

$$\int_\Omega |\varrho|\theta\wedge\beta^{n-1} < +\infty, \tag{10}$$

and where V is built so that

$$\limsup_{\varepsilon\to 0} \int_{\partial\Omega_\varepsilon} V^+\, dS_\varepsilon < +\infty.$$

Partial results were obtained by L. Gruman [13] and G. Laville [22]. In fact, Theorem 5 is a consequence of an existence theorem for the $\bar\partial$ which is especially conceived for the theory of H^p spaces. For simplicity, we only consider the (0, 1) form.

THEOREM 6. *If f is a (0, 1) current on Ω, $\bar\partial$ closed and if the coefficients of f and of the current $|\varrho|^{-1/2}d''\varrho\wedge f$ are bounded measures on Ω, then there exists $u\in L^1(\Omega)$ such that*

$$\bar\partial u = f \quad \text{in } \Omega,$$

and such that u has a boundary value in $L^1(\partial\Omega)$ in the sense of Stoke's formula:

$$\int_{\partial\Omega} u\wedge\varphi = \int_\Omega f\wedge\varphi + \int_\Omega u\wedge\bar\partial\varphi,$$

for all $(n, n-1)$ forms φ of class C^1 in $\bar\Omega$.

Theorem 6 was obtained by explicit integral formula for differential form of Cauchy–Leray–Poisson–Szegö type, which are closely related with the kernels for $\bar\partial$ already built in order to obtain L^∞ estimates for $\bar\partial$ (cf. **[26]**, **[15]**, **[20]**, **[30]**).

Theorem 5 follows from Theorem 6 using classical splitting of $\partial\bar\partial$ in d and $\bar\partial$, homotopy formula for d. The new argument is that (10) implies very strong restriction on the *tangential* coefficients of θ, for instance:

$$\int_\Omega \theta \wedge \partial_\varrho \wedge \bar\partial_\varrho \wedge (\partial\bar\partial_\varrho)^{n-2} < +\infty$$

(found in another more restrictive form by P. Malliavin **[28]**).

In another direction, Theorem 6 is closely connected with the "corona problem" for $H^\infty(\Omega)$ (i.e. the determination of the spectrum of $H^\infty(\Omega)$) and with Carleson–Hörmander measures on Ω (cf. N. Varopoulos **[42]** and **[4]**, **[38]**, **[18]**).

Other results for hypersurfaces with polynomial growth:

$$\int_X |\varrho(z)|^{\alpha+1}\, d\sigma(z) < +\infty \quad (\alpha > 0)$$

where recently in 1977 obtained by similar methods by G. M. Henkin **[6]** (cf. also **[44]**). All results of this section are in fact a consequence of an existence theorem for $\bar\partial$.

Perhaps, methods which will be more specific to the real $i\partial\bar\partial$ operator, will permit us to reach better results concerning for instance zeros of $H^p(\Omega)$. Besides, there is no result similar to Theorem 5 for analytic sets of codimension >1.

Bibliography

1. E. Bishop, *Conditions for the analyticity of certain sets*, Michigan Math. J. **11** (1964), 289—304.

2. E. Bombieri, *Algebraic values of meromorphic maps*, Invent. Math. **10** (1970), 267—287.

3. Chee Pak Song, *The Blaschke condition for bounded holomorphic function*, Trans. Amer. Math. Soc. **148** (1970), 248—263.

4. L. Carleson, *The corona theorem*, Proceedings of the 15$^{\text{th}}$ Scandinavian Congress (Oslo, 1968), Lecture Notes in Math. vol. 118, Springer-Verlag, Berlin and New York, 1970, pp. 121—132.

5. M. Cornalba and B. Shiffman, *A counter example to the "Transcendental Bezout" Problem*, Ann. of Math. **96** (1972), 402—406.

6. Daoutov and G. M. Henkin, *Zeros des fonctions holomorphes d'ordre fini, résolution et estimation pondérée de l'opérateur $\bar\partial$*, preprint (en russe), Institut de Physique Kirienskij, Krasnojarsk, 1977.

7. R. Draper, *Intersection theory in analytic geometry*, Math. Ann. **180** (1969), 175—204.

8. O. Forster und K. J. Ramspott, *Über die Darstellung analytischer Mengen*, Sb. Bayer. Akad. Wiss. Math.-Nat. Kl., 1963, 1964, 89—99.

9. ——, *Analytische Modulgarben und Eindromissbündel*, Invent. Math. **2** (1966), 145—170.

10. P. A. Griffiths and J. King, *Nevanlinna theory and holomorphic mappings between algebraic varieties*, Acta. Math. **130** (1973), 145—220.

11. P. A. Griffiths, *On the Bezout problem for entire analytic sets*, Ann. of Math. **100** (1974), 533—552.

12. L. Gruman, *The zeros of holomorphic functions in strictly pseudoconvex domains*, Trans. Amer. Math. Soc. **207** (1975), 163—174.

13. *La géométrie globale des ensembles analytiques*, Séminaire Pierre Lelong (Analyse), Année 1975/76, Lecture Notes in Math., vol. 578, Springer-Verlag, Berlin and New York, pp. 236—245.

14. R. Harvey and J. King, *On the structure of positive currents*, Invent. Math. **15** (1972), 47—52.

15. G. M. Henkin, *Integral representations of functions holomorphic in strictly pseudoconvex domain and applications to the $\bar\partial$ problem*. Mat. Sb. **82** (124), (1970), 300—308 = Math. U.S.S.R. Sb. **11** (1970), 273—281.

16. *Solutions with estimates of the H. Lewy and Poincaré-Lelong equations. The construction of functions of a Nevanlinna class with given zeros in a strictly pseudoconvex domain*, Dokl. Akad. Nauk. U.S.S.R. **224** (1975), 771—774 = Soviet Math. Dokl. **16** (1975), 1310—1314.

17. L. Hörmander, *An introduction to complex analysis in several variables*, Van Nostrand, Princeton, N. J., 1966, 2nd ed., 1973.

18. *L^p estimates for (pluri-) subharmonic functions*, Math. Scand. **20** (1967), 65—78.

19. J. King, *The currents defined by analytic varieties*, Acta. Math. **127** (1971), 185—220.

20. N. Kerzman, *Hölder and L^p estimates for solutions of $\bar\partial u=f$ in strongly pseudoconvex domains*, Comm. Pure Appl. Math. **24** (1971).

21. H. Kneser, *Zur Theorie der gebrochenen Funktionen mehrerer Veränderlicher*, Jber. Deutsch. Math.-Verein. **48** (1938), 1—28.

22. G. Laville, *Sur les diviseurs de la classe de Nevanlinna dans la boule de C^2*, C. R. Acad. Sci. Paris **281** (1975), A 145—148.

23. P. Lelong, *Intégration sur un ensemble analytique complexe*, Bull. Soc. Math. France **85**, (1957) 239—262.

24. *Fonctions entières (n variables) et fonctions plurisousharmoniques d'ordre fini dans C^n*, J. Anal. Math. Jerusalem **12** (1964), 365—407.

25. *Sur la structure des courants positifs fermés*, Séminaire Pierre Lelong (Analyse), Année 1975/76, Lecture Notes in Math., vol. 578, Springer-Verlag, Berlin and, New York, pp. 136—156.

26. I. Lieb, *Die Cauchy-Riemannschen Differentialgleichungen auf streng pseudokonvex Gebieten*. Math. Ann. **190** (1970), 6—44.

27. B. Malgrange, *Existence et approximation des solutions des équations aux dérivées partielles et équation de convolution*, Ann. Inst. Fourier **6** (1956), 271—355.

28. P. Malliavin, *Fonctions de Green d'un ouvert strictement pseudoconvexen et classe de Nevanlinna*, C. R. Acad. Sci. Paris **278** (1974), A141—144.

29. A. Martineau, *Equations différentielles d'ordre infini*, Bull. Soc. Math. France **95** (1967), 109—154.

30. N. Øvrelid, *Integral representation formulas and L^p estimates for the $\bar\partial$-equation*, Math. Scand. **29** (1971), 137—160.

31. W. Rudin, *Zeros of holomorphic functions in balls*, Nederl. Akad. Wetensch. Proc. Ser. A **79** Indag. Math., **38** (1976).

32. Y. T. Siu, *Analyticity of sets associated to Lelong numbers and the extension of closed positive currents*, Invent. Math. **27** (1974), 53—156.

33. H. Skoda, *Solution à croissance du second problème du Cousin dans C^n*, Ann. Inst. Fourier Grenoble **21** (1971), 11—23.

34. *Croissance des fonctions entières s'annulant sur une hypersurface donné de C^n*, Séminaire Pierre Lelong (Analyse), 1970/71, Lecture Notes in Math., vol. 332, Springer-Verlag, Berlin and New York.

35. *Sous-ensembles analytiques d'ordre fini ou infini dans C^n*, Bull. Soc. Math. France **100** (1972), 353—408.

36. ——— *Nouvelle méthode pour l'étude des potentiels associés aux ensembles analytiques*, Séminaire Pierre Lelong (Analyse), 1972/73, Lecture Notes in Math., vol. 410 Springer-Verlag, Berlin and New York.

37. ——— *Boundary values for the solution of the $\bar{\partial}$-equation and application to the Nevanlinna class*, Spaces of Analytic Functions (Kristiansand, Norway, 1975), Lecture Notes in Math., vol 512, Springer-Verlag, Berlin and New York 1976, pp. 166—177.

38. ——— *Valeurs au bord pour les solutions de l'opérateur d″ et caractérisation des zéros des fonctions de la classe de Nevanlinna*, Bull. Soc. Math. France **104** (1976), 225—299.

39. W. Stoll, *Ganze Funktionen endlicher Ordnung mit gegebenen Nullstellenflächen*, Math. Z. **57** (1953), 211—237.

40. ——— *The growth of the area of a transcendental analytic set*, Math. Ann. **156** (1964), 47—48, 144—170.

41. ——— *A Bezout estimate for complete intersections*, Ann. of Math. **96** (1972), 361—401.

42. N. Varopoulos, *B. M. O. functions and the equation*, Pacific. J. Math. **71** (1977), 221—273.

43. M. Waldschmidt, *Propriétés arithmétiques des fonctions des plusieurs variables* (II), Séminaire P. Lelong (Analyse), 1975/76, Lecture Notes in Math., vol. 578, Springer-Verlag, Berlin and New York.

44. N. Weyland, *Fonctions holomorphes d'ordre fini dans les domaines strictement convexes*, C. R. Acad. Sci. Paris **283** (1976), 697—699.

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