

Modular Representations of p -adic Groups and of Affine Hecke Algebras

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Abstract

I will survey some results in the theory of modular representations of a reductive p -adic group, in positive characteristic $\ell \neq p$ and $\ell = p$.

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Introduction The congruences between automorphic forms and their applications to number theory are a motivation to study the smooth representations of a reductive p -adic group G over an algebraically closed field R of *any characteristic*. The purpose of the talk is to give a survey of some aspects of the theory of R -representations of G . In positive characteristic, most results are due to the author; when proofs are available in the literature (some of them are not !), references will be given.

A prominent role is played by the unipotent block which contains the trivial representation. There is a finite list of types, such that the irreducible representations of the unipotent block are characterized by the property that they contain a unique type of the list. The types define functors from the R -representations of G to the right modules over generalized affine Hecke algebras over R with different parameters; in positive characteristic ℓ , the parameters are 0 when $\ell = p$, and roots of unity when $\ell \neq p$.

In characteristic 0 or $\ell \neq p$, for a p -adic linear group, there is a Deligne-Langlands correspondence for irreducible representations; the irreducible in the unipotent block are annihilated by a canonical ideal J ; the category of representations annihilated by J is Morita equivalent to the affine Schur algebra, and the unipotent block is annihilated by a finite power J^k .

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New phenomena appear when $\ell = p$, as the supersingular representations discovered by Barthel-Livne and classified by Ch. Breuil for $GL(2, \mathbf{Q}_p)$. The modules for the affine Hecke algebras of parameter 0 and over R of characteristic p , are more tractable than the R -representations of the group, using that the center Z of a $\mathbf{Z}[q]$ -affine Hecke algebra H of parameter q is a finitely algebra and H is a generated Z -module. The classification of the simple modules of the pro- p -Iwahori Hecke algebra of $GL(2, F)$ suggests the possibility of a Deligne-Langlands correspondence in characteristic p .

Complex case

Notation. \mathbf{C} is the field of complex numbers, $G = \underline{G}(F)$ is the group of rational points of a reductive connected group \underline{G} over a local non archimedean field F with residual field of characteristic p and of finite order q , and $\text{Mod}_{\mathbf{C}} G$ is the category of complex smooth representations of G . All representations of G will be smooth, the stabilizer of any vector is open in G . An abelian category \mathcal{C} is right (left) Morita equivalent to a ring A when \mathcal{C} is equivalent to the category of right (left) A -modules.

The modules over complex affine Hecke algebras with parameter q are related by the Borel theorem to the complex representations of reductive p -adic groups.

Borel Theorem *The unipotent block of $\text{Mod}_{\mathbf{C}} G$ is (left and right) Morita equivalent to the complex Hecke algebra of the affine Weyl group of G with parameter q .*

The proof has three main steps, in reverse chronological order, Bernstein a) [B] [BK], Borel b) [Bo], [C], Iwahori-Matusmoto c) [IM], [M].

a) **(1.a.1)** $\text{Mod}_{\mathbf{C}} G$ is a product of indecomposable abelian subcategories “the blocks”.

The unipotent block contains the trivial representation. The representations in the unipotent block will be called unipotent, although this term is already used by Lusztig in a different sense.

(1.a.2) *The irreducible unipotent representations are the irreducible subquotients of the representations parabolically induced from the unramified characters of a minimal parabolic subgroup of G .*

b) Let I be an Iwahori subgroup of G (unique modulo conjugation).

(1.b.1) *The category of complex representations of G generated by their I -invariant vectors is abelian, equivalent by the functor*

$$V \mapsto V^I = \text{Hom}_{\mathbf{C}G}(\mathbf{C}[I \backslash G], V)$$

to the category $\text{Mod } H_{\mathbf{C}}(G, I)$ of right modules of the Iwahori Hecke algebra

$$H_{\mathbf{C}}(G, I) = \text{End}_{\mathbf{C}G} \mathbf{C}[I \backslash G].$$

(1.b.2) *This abelian category is the unipotent block.*

c) (1.c) *The Iwahori Hecke algebra $H_{\mathbf{C}}(G, I)$ is the complex Hecke algebra of the affine Weyl group of G with parameter q .*

The algebra has a very useful description called the Bernstein decomposition [L1] [BK], basic for the geometric description of Kazhdan-Lusztig [KL].

From (1.b.1), the irreducible unipotent complex representations of G are in natural bijection with the simple modules of the complex Hecke algebra $H_{\mathbf{C}}(G, I)$. By the “unipotent” Deligne-Langlands correspondence, the simple $H_{\mathbf{C}}(G, I)$ -modules “correspond” to the G' -conjugacy classes of pairs (s, N) , where $s \in G'$ is semisimple, $N \in \text{Lie } G'$ and $\text{Ad}(s)N = qN$, where G' is the complex dual group of G with Lie algebra $\text{Lie } G'$. This is known to be a bijection when $G = GL(n, F)$ [Z] [R]. When G is adjoint and unramified (quasi-split and split over a finite unramified extension), it is also known to be a bijection if one adds a third ingredient, a certain irreducible geometric representation ρ of the component group of the simultaneous centralizer of both s and N in G' ; this was done by Chriss [C], starting from the basic case where G is split of connected center treated by Kazhdan Lusztig [KL] and by Ginsburg [CG] *. The adjoint and unramified case is sufficient for many applications to automorphic forms; to my knowledge the general case has not been done.

According to R. Howe, the complex blocks should be parametrized by types. The basic type, the trivial representation of an Iwahori subgroup, is the type of the unipotent block. An arbitrary block should be right Morita equivalent to the Hecke algebra of the corresponding type. The Hecke algebra of the type should be a generalized affine Hecke complex algebra with different parameters equal to positive powers of p . This long program started in 1976 is expected to be completed soon. The most important results are those of Bushnell-Kutzko for $GL(n, F)$ [BK], of Morris for the description of the Hecke algebra of a type [M], of Moy and Prasad for the definition of unrefined types [MP].

Conjecturally, the classification of simple modules over complex generalized affine Hecke algebras and the theory of types will give the classification of the complex irreducible representations of the reductive p -adic groups.

We consider now the basic example, the general linear p -adic group $GL(n, F)$. The the complex irreducible representations of $GL(n, F)$ over R are related by the “semi-simple” Deligne-Langlands correspondence (proved by Harris-Taylor [HT1]

* Introduction page 18. Complex representations of the absolute Weil-Deligne group with semi-simple part trivial on the *inertia subgroup* (6.1) are in natural bijection with the ℓ -adic representations of the absolute Weil group trivial on the *wild ramification subgroup* for any prime number $\ell \neq p$ [T] [D]. In the Deligne-Langlands correspondence, one considers only the representations which are Frobenius semi-simple.

and Henniart [He]), to the representations of the Galois group $\text{Gal}(\overline{F}/F)$ of a separable algebraic closure \overline{F} of F .

Deligne-Langlands correspondence

(1.d) *The blocks of $\text{Mod}_{\mathbf{C}} GL(n, F)$ are parametrized by the conjugacy classes of the semi-simple n -dimensional complex representations τ of the inertia group $I(\overline{F}/F)$ which extend to the Galois group $\text{Gal}(\overline{F}/F)$.*

(1.e) *The block parametrized by τ is equivalent to the unipotent block of a product of linear groups $G_\tau = GL(d_1, F_1) \times \dots \times GL(d_r, F_r)$ over unramified extensions F_i of F where $\sum d_i[F_i : F] = n$.*

(1.f) *The irreducible unipotent representations of $GL(n, F)$ are parametrized by the $GL(n, R)$ -conjugacy classes of pairs (s, N) where $s \in GL(n, \mathbf{C})$ is semi-simple, $N \in M(n, \mathbf{C})$ is nilpotent, and $sN = qNs$.*

Modular case Let R be an algebraically closed field of any characteristic. When the characteristic of R is 0, the theory of representations of G is essentially like the complex theory, and the above results remain true although some proofs need to be modified and this is not always in the literature. From now on, we will consider “modular or mod ℓ ” representations, i.e. representations over R of characteristic $\ell > 0$.

Banal primes Although a reductive p -adic group G is infinite, it behaves often as a finite group. Given a property of complex representations of G which has formally a meaning for mod ℓ representations of G , one can usually prove that outside a finite set of primes ℓ , the property remains valid. This set of primes is called “banal” for the given property.

For mod ℓ representations the Borel theorem is false, because the mod ℓ unipotent block of $GL(2, F)$ contains representations without Iwahori invariant vectors when $q \equiv -1 \pmod{\ell}$ [V1].

(2) *The Borel theorem is valid for mod ℓ representations when ℓ does not divide the pro-order of any open compact subgroup of G .*

These primes are banal for the three main steps in the proof of the complex theorem.

a) (2.a) *Any prime is banal for the decomposition of $\text{Mod}_R G$ in blocks.*

The complex proof of (1.a.1) does not extend. There is a new proof relying of the theory of unrefined types [V5 III.6] when $\ell \neq p$.

b) (1.a.2), (1.b.1), (1.b.2) remain true because ℓ does not divide the pro-order of the Iwahori subgroup I and [V2] [V4]:

(2.b) *Any irreducible cuspidal mod ℓ -representation of G is injective and projective in the category of mod ℓ -representations of G with a given central character when ℓ is as in theorem 2.*

c) Any prime ℓ is banal for the Iwahori-Matsumoto step because the proofs of Iwahori-Matsumoto and of Morris are valid over \mathbf{Z} , and for any commutative ring A , the Iwahori Hecke A -algebra

$$H_A(G, I) = \text{End}_{AG} A[I \backslash G] \simeq H_{\mathbf{Z}}(G, I) \otimes_{\mathbf{Z}} A$$

is isomorphic to the Hecke A -algebra of the affine Weyl group of G with parameter q_A where q_A is the natural image of q in A .

The primes ℓ of theorem 2 are often called *the banal primes of G* because such primes are banal for many properties. For example, the category of mod ℓ -representations of G with a given central character has finite cohomological dimension [V4]. In the basic example $GL(n, F)$, ℓ is banal when $\ell \neq p$ and the multiplicative order of q modulo ℓ is $> n$.

Limit primes The set of primes banal for (1.a.2), (1.b.1) is usually larger than the set of banal primes of G . The primes of this set which are not banal will be called, following Harris, the *limit primes* of G . In the basic example $GL(n, F)$, the limit primes ℓ satisfy $q \equiv 1 \pmod{\ell}$ and $\ell > n$ [V3]. For number theoretic reasons, the limit primes are quite important [DT] [Be] [HT2]. They satisfy almost all the properties of the banal primes. For linear groups, the limit primes are banal for the property that no cuspidal representation is a subquotient of a proper parabolically induced representation. This may be true for G general.

Let $\overline{\mathbf{Q}}_\ell$ be an algebraic closure of the field \mathbf{Q}_ℓ of ℓ -adic numbers, $\overline{\mathbf{Z}}_\ell$ its ring of integers and $\overline{\mathbf{F}}_\ell$ its residue field. The following statements follow from the theory of types, or from the description of the center of the category of mod ℓ representations (the Bernstein center).

(3.1) *The reduction gives a surjective map from the isomorphism classes of the irreducible cuspidal integral $\overline{\mathbf{Q}}_\ell$ -representations of G to the irreducible cuspidal $\overline{\mathbf{F}}_\ell$ -representations of G , when ℓ is a banal or a limit prime for G .*

Natural characteristic The interesting case where the characteristic of R is p is not yet understood. There is a simplification: R -representations of G have non zero vectors invariant by the pro- p -radical I_p of I . The irreducible are quotients of $R[I_p \backslash G]$.

Some calculations have been made for $GL(2, F)$ [BL] [Br] [V9]. A direct classification of the irreducible R -representations of $G = GL(2, \mathbf{Q}_p)$ [BL] [Br] and of the pro- p -Iwahori Hecke R -algebra $H_R(G, I_p) = \text{End}_{RG} R[I_p \backslash G]$ (called a mod p pro- p -Iwahori Hecke algebra) shows:

(4.1) *Suppose R of characteristic p . The pro- p -Iwahori functor gives a bijection between the irreducible R -representations of $GL(2, \mathbf{Q}_p)$ and the simple right $H_R(G, I_p)$ -modules.*

This is the “mod p simple Borel theorem” for the pro- p -Iwahori group of

$GL(2, \mathbf{Q}_p)$. In particular p is banal for the simple version of (1.b.1) when $G = GL(2, \mathbf{Q}_p)$. Irreducible mod p representations of $GL(2, F)$ which are non subquotients of parabolically induced representations from a character of the diagonal torus are called supersingular [BL]. There is a similar definition for the mod p simple supersingular modules of the pro- p -Iwahori Hecke algebra of $GL(2, F)$.

(4.2) *There is a natural bijection between the mod p simple supersingular modules of the mod p pro- p -Iwahori Hecke algebra of $GL(2, F)$ and the mod p irreducible dimension 2 representations of the absolute Weil group of F .*

This suggests the existence of a mod p Deligne-Langlands correspondence. Some computations are being made by R. Ollivier for $GL(3, F)$.

We end this section with a new result on affine Hecke algebras as in [L3], which is important for the theory of representations modulo p .

(4.3) *Let H be an affine Hecke $\mathbf{Z}[q]$ -algebra of parameter q associated to a generalized affine Weyl group W . Then the center Z of H is a finitely generated $\mathbf{Z}[q]$ -algebra and H is a finitely generated Z -module.*

The key is to prove that H has a $\mathbf{Z}[q]$ -basis $(q^{k(w)} E_w)_{w \in W}$ where (E_w) is a Bernstein $\mathbf{Z}[q^{-1}]$ -basis of $H[q^{-1}]$. The assertion (4.3) was known when the parameter q is invertible.

Non natural characteristic R an algebraically closed field of positive characteristic $\ell \neq p$. Any prime $\ell \neq p$ is banal for the “simple Borel theorem”. The “simple Borel theorem” is true mod $\ell \neq p$.

(5.1) *Suppose $\ell \neq p$. The Iwahori-invariant functor gives a bijection between the irreducible R -representations of G with $V^I \neq 0$ and the simple right $H_R(G, I)$ -modules.*

The existence of an Haar measure on G with values in R implies that $\text{Mod}_R G$ is left Morita equivalent to the convolution algebra $H_R(G)$ of locally constant, compact distributions on G with values in R . When the pro-order of I is invertible in R , the Haar measure on G over R normalized by I is an idempotent of $H_R(G)$, and (5.1) could have been already proved by I. Schur [V3]. In general (5.1) follows from the fact that $R[I \backslash G]$ is “almost projective” [V5].

More generally, one expects that the Howe philosophy of types remains true for modular *irreducible* representations. Their classification should reduce to the classification of the simple modules for generalized affine Hecke R -algebras of parameters equal to 0 if $\ell = p$, and to roots of unity if $\ell \neq p$. This is known for linear groups if $\ell \neq p$ [V5] or in characteristic $\ell = p$ for $GL(2, F)$ [V9].

The unipotent block is described by a finite set S of modular types, the “unipotent types” [V7]. The set S contains the class of the basic type (I, id) . In the banal or limit case, this is the only element of S . A unipotent type (P, τ) is the G -

conjugacy class of an irreducible R -representation of a parahoric subgroup P of G , trivial on the pro- p -radical P_p , cuspidal as a representation of P/P_p (the group of rational points of a finite reductive group over the residual field of F). The isomorphism class of the compactly induced representation $\text{ind}_P^G \tau$ of G determines the G -conjugacy class of τ , and conversely. We have $\text{ind}_I^G \text{id} = R[I \backslash G]$.

(5.2) Theorem *Suppose $\ell \neq p$. There exists a finite set S of types, such that*

- $\text{ind}_P^G \tau$ is unipotent for any $(P, \tau) \in S$,
- an irreducible unipotent R -representation V of G is a quotient of $\text{ind}_P^G \tau$ for a unique $(P, \tau) \in S$, called the type of V ,
- the map $V \mapsto \text{Hom}_{RG}(\text{ind}_P^G \tau, V)$ between the irreducible quotients of $\text{ind}_P^G \tau$ and the right $H_R(G, \tau) = \text{End}_{RG} \text{ind}_P^G \tau$ modules is a bijection.

The set S has been explicitly described only when G is a linear group [V5]. In the example of $GL(2, F)$ and $q \equiv -1$ modulo ℓ , the set S has two elements, the basic class and the class of $(GL(2, O_F), \tau)$ where τ is the cuspidal representation of dimension $q-1$ contained in the reduction modulo ℓ of the Steinberg representation of the finite group $GL(2, \mathbf{F}_q)$.

The Hecke algebra $H_R(G, \tau)$ of the type (P, τ) could probably be described a generalized affine Hecke R -algebra with different parameters (complex case [M] [L2], modular case for a finite group [GHM]).

The linear group in the non natural characteristic We consider the basic example $G = GL(n, F)$ and R an algebraically closed field of positive characteristic $\ell \neq p$.

(6.1) *Any prime $\ell \neq p$ is banal for the Deligne-Langlands correspondence.*

This means that (1.d) (1.e) (1.f) remain true when \mathbf{C} is replaced by R . The proof is done by constructing congruences between automorphic representations for unitary groups of compact type [V6].

The unipotent block is partially described by the *affine Schur algebra*

$$S_R(G, I) = \text{End}_{RG} V, \quad V = \oplus_{P \supset I} \text{ind}_P^G \text{id},$$

which is the ring of endomorphisms of the direct sum of the representations of G compactly induced from the trivial representation of the parahoric subgroups P containing the Iwahori subgroup I . The functor of I -invariants gives an isomorphism from the endomorphism ring of the RG -module V to the endomorphism ring of the right $H_R(G, I)$ -module V^I and the $(S_R(G, I), H_R(G, I))$ module V^I satisfies the double centralizer property [V8].

$$(6.2) \quad \text{End}_{H_R(G, I)} V^I = S_R(G, I), \quad \text{End}_{S_R(G, I)} V^I = H_R(G, I).$$

In the complex case, the affine Schur algebra $S_{\mathbf{C}}(G, I)$ is isomorphic to an algebra already defined R.M. Green [Gr]: A complex *affine quantum linear group*

$\hat{U}(gl(n, q))$ has a remarkable representation W of countable dimension such that the tensor space $W^{\otimes n}$ satisfies the double centralizer property

$$\text{End}_{\hat{S}(n, q)} W^{\otimes n} = \hat{H}(n, q), \quad \text{End}_{\hat{H}(n, q)} W^{\otimes n} = \hat{S}(n, q)$$

where $\hat{S}(n, q)$ is the image of the action of $\hat{U}(gl(n, q))$ in $W^{\otimes n}$. The algebras $\hat{S}(n, q)$ and $\hat{H}(n, q)$ are respectively isomorphic to $S_{\mathbf{C}}(G, I)$ and $H_{\mathbf{C}}(G, I)$; the bimodules $W^{\otimes n}$ and V^I are isomorphic.

Let J be the annihilator of $R[I \setminus G]$ in the global Hecke algebra $H_R(G)$.

(6.3) Theorem Suppose $\ell \neq p$.

There exists an integer $k > 0$ such that the unipotent block of $\text{Mod}_R G$ is the set of R -representations of G annihilated by J^k .

An irreducible representation of G is unipotent if and only if it is a subquotient of $R[I \setminus G]$, if and only if it is annihilated by J .

The abelian subcategory of representations of G annihilated by J is Morita equivalent to the affine Schur algebra $S_R(G, I)$.

This generalizes the Borel theorem to mod ℓ representations when G is a linear group. The affine Schur algebra exists and the double centralizer property (6.2) is true for a general reductive p -adic group G ; in the banal case, the affine Schur algebra is Morita equivalent to the affine Hecke algebra.

Integral structures Let ℓ be any prime number. There are two notions of integrality for an admissible $\overline{\mathbf{Q}}_\ell$ -representation V of G , $\dim V^K < \infty$ for all open compact subgroups K of G , which coincide when $\ell \neq p$ [V3]. One says that V is integral if V contains a G -stable $\overline{\mathbf{Z}}_\ell$ -submodule generated by a $\overline{\mathbf{Q}}_\ell$ -basis of V , and V is locally integral if the $H_{\overline{\mathbf{Q}}_\ell}(G, K)$ -module V^K is integral, i.e. contains a $H_{\overline{\mathbf{Z}}_\ell}(G, K)$ -submodule $\overline{\mathbf{Z}}_\ell$ -generated by a $\overline{\mathbf{Q}}_\ell$ -basis of V^K , for all K .

When V is irreducible and integral, the action of the center Z of G on V , the central character, is integral, i.e. takes values in $\overline{\mathbf{Z}}_\ell$. The situation is similar for a simple integral $H_{\overline{\mathbf{Q}}_\ell}(G, I)$ -module W . The central character is integral, i.e. its restriction to the center of $H_{\overline{\mathbf{Z}}_\ell}(G, I)$ takes values in $\overline{\mathbf{Z}}_\ell$.

(7.1) Théorème

a) An irreducible cuspidal $\overline{\mathbf{Q}}_\ell$ -representation V of G is integral if and only if its central character is integral.

b) A simple $H_{\overline{\mathbf{Q}}_\ell}(G, I)$ -module is integral if and only if its central character is integral.

c) An irreducible representation V of G with $V^I \neq 0$ is locally integral if and only if V^I is an integral $H_{\overline{\mathbf{Q}}_\ell}(G, I)$ -module.

The assertion b) results from (4.3). For a) [V3]. For $\ell = p$, c) is due to J.-F. Dat, using its theory of ℓ -adic analysis [D].

A general irreducible $\overline{\mathbf{Q}}_\ell$ -representation V of G is contained in a parabolically induced representation of an irreducible cuspidal representation W of a Levi subgroup of G . If W is integral then V is integral, but the converse is false when $\ell = p$. When $\ell \neq p$, the converse is proved for classical groups by Dat using results of Mœglin (there is a gap in the “proof” of the converse in [V3]).

(7.2) Brauer-Nesbitt principle [V3][V11] *When $\ell \neq p$, the integral structures L of an irreducible $\overline{\mathbf{Q}}_\ell$ -representation of G are $\overline{\mathbf{Z}}_\ell G$ -finitely generated (hence commensurable) and their reduction $L \otimes \overline{\mathbf{F}}_\ell$ are finite length $\overline{\mathbf{F}}_\ell$ -representations of G with the same semi-simplification (modulo isomorphism).*

When $\ell = p$, this is false. An integral cuspidal irreducible $\overline{\mathbf{Q}}_p$ -representation V of G embeds in $\overline{\mathbf{Q}}_p[\Gamma \backslash G]$, for any discrete co-compact-mod-center subgroup Γ of G , and has a natural integral structure with an admissible reduction [V10]. When the theory of types is known, V is induced from an open compact-mod-center subgroup, hence has an integral structure with a non admissible reduction, which is not commensurable with the first one.

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