55 D. Aubin, "The Withering Immortality of Nicolas Bourbaki: A Cultural Connector at the Confluence of Mathematics, Structuralism, and the Oulipo in France," *Science in Context* 10 (1997): 297–342; M. Armatte, "Mathématiques 'modernes' et sciences humaines," in B. Belhoste, H. Gispert, and N. Hulin (eds.), *Les Sciences au lycée* (Paris: Vuibert, 1996), 77–88; J-P. Kahane, "Les mathématiques, hier et demain," in *Les Sciences au lycée*, 89–98.

56 The first occurrence of this expression was in a letter in French from Jacobi to Legendre, 2 July 1830; see C. Jacobi, *Gesammelte Werke* (Berlin: Reimer, 1884), 1:454. It reappears in André Weil, "L'avenir des mathématiques," written in 1943 and published in Le Lionnais (ed.), *Les grands courants*. In the 1960s and 1970s, it was often used by various authors and taken up by Jean Dieudonné as the title of his bestseller, *Pour l'honneur de l'esprit humain. Les mathématiques aujourd'hui* (Paris: Hachette, 1987).

57 In the United States, see in particular the efforts of the American Mathematical Society following the publication of the David report and in France, a conference jointly organized in 1985 by the Société mathématique de France and the Société de mathématiques appliquées et industrielles.

58 Cf. David Mumford, "Trends in the Profession of Mathematics," *Berlin Intelligencer*, International Congress of Mathematicians, Berlin, August 1998.

3 Forms of explanation in the catastrophe theory of René Thom: topology, morphogenesis, and structuralism

David Aubin

Voici maintenant qu'après l'âge des denrées et des matières, après celui de l'énergie, nous avons commencé à vivre celui de la forme.—Pierre Auger, *Proceedings of the First International Conference on Cybernetics*

Peut-être personne n'est plus capable que le mathématicien de suivre une question de forme pure.—George D. Birkhoff, "Quelques éléments mathématiques de l'art"

"Science, some say, is in crisis." When the French mathematician René Thom wrote this in 1975, the catastrophe theory he had imagined during the previous decade had already made him famous. Thom nonetheless agreed that science faced an unprecedented crisis. Decrease of governmental support, students' disaffection from scientific careers, accumulation of trash, and poisoning of the earth: albeit highly visible, these signs merely pointed to a deeper malaise within science itself. Indeed, hidden behind triumphant proclamations of progress and success, Thom saw a "manifest stagnation of scientific thought vis-à-vis the central problems affecting our knowledge of reality." At bottom, he contended, this stagnation was due to the fact that "science [had sunk] into the futile hope of exhaustively describing reality, while forbidding itself to 'understand' it."[1]

Understanding—this was science's "prime vocation," and the way out of the present crisis. Inspired by Kuhnian theses, Thom believed the crisis presaged an important paradigm shift. Science "must come back to this essential goal [which is] to understand reality, to *simulate nature*. . . . If, as I wish to believe, this necessary mutation is to be accomplished, will we not then be able to say of science that it remains man's hope?"[2] The only solution to the problem of contemporary science was more science. This would not be the old science, but a new one, which would endeavor to provide explanations rather than mere descriptions or predictions.

His own catastrophe theory, of course, was for Thom a prime example of this new type of science.[3] Slowly appreciated when it was introduced in the late 1960s, catastrophe theory was propelled on a wave of hype and enthusiasm during the mid-1970s only to die out in bitter controversies by the end of the decade. Caught in fierce debates, the movement nearly vanished from the scene of science. True, the theorems that Thom and his collaborators proved, have survived as "a beautiful, intriguing field of pure mathematics."[4] Even the concepts they introduced lived on in other guises, as Thom was clearly aware: "Sociologically speaking, it can be said that this theory is a shipwreck. But in some sense, it is a subtle wreck, because the ideas that I have introduced gained ground. In fact, they are now incorporated in everyday language. . . . The notions [of catastrophe theory] have become part of the ordinary baggage of modelers. Therefore, it is true that, in a sense, the *ambitions* of the theory failed, but in *practice*, the theory has succeeded."[5]

This is especially true of chaos theory, with which catastrophe theory had important interactions. Both inspired by topology, these theories shared some of their mathematical concepts, their modeling practices, their practitioners and institutional locations, their modes of explanation, and their general aims. Only in the second half of the 1970s did they definitely part from each other.[6]

Moreover, catastrophe theory offered types of explanation that were directly inspired not only by mathematics, but also by biology and structuralism. The new explanations were perceived as subverting dominant ideologies in all of these disciplines. Centered on problems of structure and form, they aimed at providing accounts for the emergence and destruction of morphologies, based not on underlying forces but on mathematical principles.[7] "Thus, we are catching a glimpse of the possibility of creating a dynamic structuralism," Thom declared, while proposing explanations that grew out of his interest in embryology.[8] We shall see, for example, how the important concept of an "attractor" emerged from his understanding of embryologist Conrad Hal Waddington's epigenetic landscapes.

Finally, because of its interdisciplinary character, catastrophe theory was, for people with widely differing agendas, a cultural connector linking mathematics, biology, the social sciences, and philosophy. It represented an incomplete transition from explanations in terms of a few simple, stable forms to an understanding of nature in terms of the complex, the fluid, and the multiple. Indeed, it may be possible to see catastrophe theory as a crucial transitional stage from structuralism to poststructuralism, perhaps even from modernism to postmodernism.[9]

Although the global ambitions of catastrophe theory dimmed markedly in the late 1970s, it remains of interest to determine what has survived "in practice." We can begin by understanding the specific context from which catastrophe theory emerged. More specifically, we need to reconstitute and

reinterpret the type of explanations Thom proposed. Today his project is often obscured by the settlement that took place in the latter part of the 1970s, effectively dividing the world between stable and chaotic systems in the course of opening new paths for understanding the sources of complexity in the world.[10]

This essay will explore the new forms of scientific explanation that Thom offered as an alternative to what he denigrated as the traditional "reductionist approach." Indeed, his philosophy of science could be summarized as follows: first one classifies a phenomenology by describing its morphologies, then one strives for explanations, which, as Thom believed, could be achieved by following one of two philosophically distinct approaches, the *reductionist* or the *structural*. The former accounted for morphogenesis in terms of other morphologies; the latter eschewed such attempts and looked for autonomous, intrinsic explanations that did not depend on other levels of reality.

None of these steps is self-evident. Their actualization depends on the modeling practices that scientists have deployed. Indeed, morphologies, like explanations, are not imposed on observers, but fashioned by the lens they choose to wear. This is what is meant by modeling practices: the actual processes by which scientists transform, using some specific means, a given raw material, selected by them, into a product which they hope will be considered knowledge about natural phenomena.[11] Together with other mathematicians—Ralph Abraham, Steve Smale, and Christopher Zeeman, who often visited him at the Institut des hautes études scientifiques (IHÉS) in Bures-sur-Yvette, France—René Thom proposed radically new modeling practices. With catastrophe theory, he wished to redefine what it meant to build a mathematical model. His experience in mathematics suggested new means of knowledge production. His forays into embryology shaped his views on what should be sound raw material for modeling. And he found in the French intellectual context—in structuralism—an inspiration for his interpretation of product-knowledge. In each case (mathematics, embryology, and structuralism), Thom used well established technical achievements to go beyond and subvert the original framework to which they belonged. The final part of this essay will examine Thom's philosophy of science as he described it around 1975, that is, after the main tenets of catastrophe theory had been well publicized but just before harsh critiques and the emergence of chaos made him retreat deeper into philosophy.

Sociologically speaking, a mathematician

A bold and comprehensive theory aimed at explaining the dynamics of shapes in the everyday world, catastrophe theory has often been narrowly construed as a mathematical approach able to deal with abrupt, discontinuous changes in nature—a rubber band that breaks, for example. For Thom, however, it was always much more than this.

What was catastrophe theory?

From 1964 to 1968, on his own account, Thom worked on an ambitious book, a manifesto, titled *Structural Stability and Morphogenesis*, which was not published until 1972, due to its publisher's financial trouble.[12] For this reason, catastrophe theory was first presented in two articles, both published in 1968. To the proceedings of a theoretical biology symposium, Thom contributed "A Dynamical Theory of Morphogenesis," and for the French journal *L'Âge de la science*, he wrote "Topology and Meaning."[13] The first article was concerned with biology, the second with semiotics. Not content with introducing a new mathematical language and exploring its consequences in some areas of science, Thom also conceived of his book, and both of these articles, as exposés of an original philosophy of science, indeed a true "natural philosophy."[14] The subtitle of his book, "An Outline of a General Theory of Models," revealed the extent of his ambitions.

A striking paradox raised by Thom may illustrate his epistemological concerns.[15] Consider an eroding cliff and the developing egg of a frog. In the former case, suppose that later microclimatic conditions and the geological nature of the soil are known, then knowledge of the physical and chemical forces at play will be excellent. Nevertheless, it is impossible to predict the future shape of the cliff. As for the egg, Thom contended, although knowledge of the substrate and developmental mechanisms is sketchy, we can still be pretty sure that it will end up as a frog! In his view, this paradox showed that blind reliance on reductionist arguments obscured problems of forms. Clearly, a new method was needed that would focus on shapes, account for their stability, and explain their creation and destruction.

For Thom, catastrophe theory supplied this method. In summary, its goal was to understand natural phenomena by approaching them directly, rather than relying on traditional reductionism. Its main concern was the creation and destruction of forms, but more precisely, as they arise at the mundane level of everyday life. Catastrophe theory posited the existence of a mathematically defined structure responsible for the stability of these forms, which he called the logos of the form. Consequently, he rejected the idea that the universe was governed by chaos or chance. The models built with the help of catastrophe theory were inherently qualitative—not quantitative—which meant that they were not suited for action or prediction, but rather aimed at describing, and intelligibly understanding, natural phenomena. Finally, Thom recognized that catastrophe theory was not a

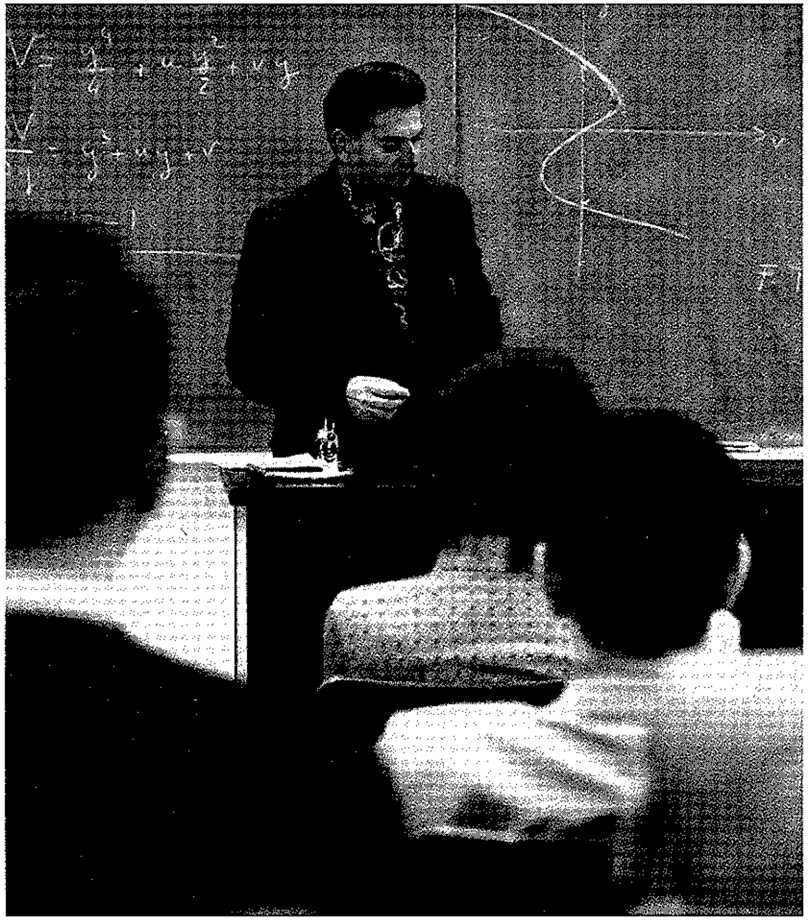
proper scientific theory, but rather a method or a language that could not be tested experimentally and therefore was not falsifiable in the sense of Karl Popper.

Mathematical styles: Bourbaki against intuition

At the source of catastrophe theory, we find a man who still "sociologically" defines himself as a mathematician. Born in 1923, René Thom recalls a "decisive encounter with Euclidean geometry" during his *lycée* years, when he fell for "the geometric mode of thought and type of proof."[16] However, his geometric, intuitive vision of mathematics was opposed to the dominant trend. In 1943, Thom experienced at the École normale supérieure "the excitement born with Bourbakist ideas."[17] Some of the Bourbaki group, already important members of the French mathematical community, were among Thom's professors. Bourbaki "was a symbol . . . of the triumph of abstraction over application, of formalism over intuition."[18] The Bourbaki group did not reject geometry as much as the intuitive approach to Euclidean geometry, upon which Thom's mathematical intuition and philosophy were built. Thom's opinion of Bourbaki was thus quite ambivalent. As one of Bourbaki's most successful students, Thom praised his introduction into France of the mathematics of Göttingen. But as David Hilbert himself once wrote, two tendencies were present in mathematics: "On the one hand, the tendency toward *abstraction*, [seeking] to crystallize the *logical* relation inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward *intuitive understanding*, [fostering] a more immediate grasp of the objects one studies, a live *rapport* with them, so to speak."[19]

For Thom, Bourbaki had clearly chosen the first path, thus failing to keep Hilbert's mathematics alive. "It is a bit as if, at the time of Vesaleus, when the method of dissection eventually imposed itself, one had wanted to identify the study of human beings with the analysis of cadavers."[20] Bourbaki's ascetic formalism killed mathematics.

Thom knew Bourbaki very well. He was once almost recruited by them but says that he literally fell asleep during the lectures.[21] Nevertheless, he was learning. Thom's early achievement was to reconcile his powerful geometric intuition with Bourbaki's arsenal. In 1946, he moved to Strasbourg with his mentor Henri Cartan, who had oriented him toward differential topology. This, in part, motivated Thom's ambiguous assessment of Bourbaki. Multidimensional spaces—which topologists mostly study—are difficult to visualize. So, a systematic, formal mode of thought, however boring and counterintuitive it might be, is then incomparably useful.



1. René Thom lecturing on catastrophe theory at Memorial University, Canada, in the early 1970s, with a section of the cusp behind him. Source: Photo Section, ETV Centre, Memorial University.

Indeed, Thom mastered the techniques offered by Bourbaki's edifice well enough to obtain results, which, according to Jean Dieudonné, marked "the modern rise of differential topology."[22] In 1958, he was awarded the highest distinction for a mathematician—the Fields Medal.

On this occasion, Heinz Hopf identified Thom's strengths. This was a time when topology was in a "stage of vigorous . . . algebraicization." Not only had algebra been found to provide "a means to treat topological problems," but also "it rather appears that most of [these] problems themselves possess an explicitly algebraic side." Still, for Hopf, there lurked the danger of "totally ignoring the geometrical content of topological problems. "In regard to this danger, I find that Thom's accomplishments have

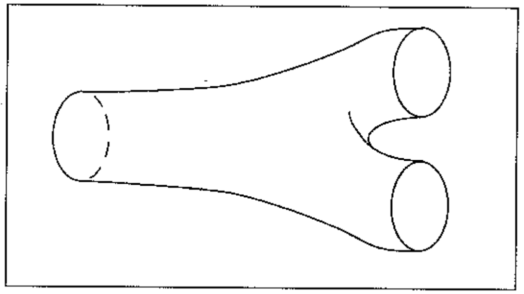
something that is extraordinarily encouraging and pleasing. While Thom masters and naturally uses modern mathematical methods and while he sees the algebraic side of his problems, his fundamental ideas . . . are of a perfectly geometric-*anschaulich* nature."[23] Thom was able to use Bourbakist algebraic methods in solving topological problems without losing sight of their anschaulich, or intuitive, character. Tim Poston, a later catastrophist, vividly contrasted Thom's style with a traditional approach *à la* Bourbaki. "Some mathematicians go at their work like engineers building a six-lane highway through the jungle, laying out surveying lines, clearing the underbrush, and so on. But Thom is like some creature of the mathematical jungle, blazing a trail and leaving just a few marks on his way to the next beautiful clearing."[24] Indeed, Thom came to view rigor in mathematics as counterintuitive and counterproductive. "Absolute rigor is only possible in and by insignificance." True to his preference for meaningful wholes over insignificant details, he held that rigor hid the essential. In mathematical research, it should always come second. "Rigor, in mathematics, is essentially a question of housekeeping [*intendance*]."[25]

Mathematical interlude I: Thom's cobordism theory

Thom's work on cobordism, for which he was awarded the Fields Medal, clearly illustrates his intuitive approach as allied with the profound knowledge of Bourbakist methods that guided most of his mathematical work.[26] As Hopf testified, cobordism was important because of the way it mixed topological and algebraic approaches in the classification of manifolds. In the following, the definition of a few concepts will be recalled. Briefly, Thom's cobordism theory enabled him to construct groups Ω^n out of equivalence classes of manifolds of dimension n, and to classify these groups.

Topology is a generalization of geometry that studies spaces with the degree of generality appropriate to a specific problem. One central concern of topology is to study the properties of spaces that do not change under a continuous transformation, that is, translation, rotation, and stretching without tearing. One such property is expressed by the concept of *dimension*: a curve is one-dimensional; a surface has two dimensions; ordinary space, three; and the space-time of general relativity, four.

Mathematicians faced with the problem of characterizing a space locally isomorphic to a Euclidean space use the notion of *manifold*. An n-dimensional manifold is a space M, such that a neighborhood V exists around each point p of M in one-to-one correspondence with a subset W of the n-dimensional Euclidean space $\mathbf{R}^n$. The study of manifolds is called differential geometry, and the classification of all manifolds of a given dimension is an important problem of topology. It is also possible to define



2. The manifold composed of two circles is cobording with the manifold consisting of a single circle because there is a "pant-shaped" smooth surface joining them. Since this is true for manifolds combining any number of circles, the group Ω^1 is the one-element trivial group.

manifolds with edges. If the manifold with edges has $n+1$ dimensions, then the edges are n-dimensional manifolds. For example, a sheet of paper rolled into a cylinder has two circles as edges. A manifold with three circles as edges is pictured in figure 2.

Let us also define equivalence relations and equivalence classes. An equivalence relation, symbolized by $\sim$, over a set S is defined so that, for all a, b, and c in S, the three following properties are satisfied: (1) reflexivity: $a \sim a$; (2) symmetry: if $a \sim b$ then $b \sim a$; and (3) transitivity: if $a \sim b$ and $b \sim c$, then $a \sim c$. The equivalence class $[a]$ of an element a of S is the subset of S that contains all the elements b that are equivalent to a, that is, all b's in S such that $b \sim a$.

Thom defined two manifolds M and N, both of dimension n, to be cobording (in French, *cobordantes*, from *bord* or "edge") if there was a manifold P of dimension $n+1$ so that M and N formed its edge. He then showed that cobording manifolds formed an equivalence class. For example, one circle is cobording with the manifolds formed by the nonintersecting union of two circles, because it is possible to unite them with a two-dimensional manifold with edges (figure 2).

Thom realized that the set Ω^n of all these equivalence classes formed a group, the group operation being defined as the nonintersecting union of manifolds. Exploiting modern formalism with the help of Jean-Pierre Serre, Thom identified the structure of those groups. He found that

$$\Omega^0=Z;\ \Omega^1=\Omega^2=\Omega^3=0;\ \Omega^4=Z;\ \Omega^5=Z_2;\ \Omega^6=\Omega^7=0.$$

(He also provided partial results for higher dimensions.)

It is worthwhile to note that if M is cobording with N, then it is possible for M to evolve in time and become N. Thus cobordism can be seen as the study of possible continuous transformations of a given shape. Retrospectively, Thom also saw it this way: "The problem of cobordism . . . is of knowing when two manifolds can be deformed one into the other without encountering a singularity in the resulting space, at any *moment* in this deformation."[27] The example of a circle becoming two circles can, very crudely of course, model cell division (figure 2).

The mathematical background of catastrophe theory

When Thom moved to Strasbourg in 1946, it hardly corresponded to the provincial exile that successful French professors often had to endure before they could trek back to Paris. In addition to the presence there of Thom's thesis director Henri Cartan, the Bourbakist Charles Ehresmann directed a topology seminar, where in 1950 Thom heard Hassler Whitney describe his work on singularities of mappings from the plane to the plane.[28] Thom also became acquainted with Morse theory concerning the relation between the topology of spaces and the singularities of real functions defined on them.

From his stay in Strasbourg, Thom drew resources congenial to his attack on the problems of singularity theory, which he founded with Morse and Whitney. Just like "living beings," Paul Montel wrote in 1930, "functions are characterized by their singularities."[29] Trying to make sense of multidimensional spaces, Thom considered singular points a blessing. He once discussed "a philosophical aspect" motivating the emphasis put on singularities, thus revealing his topological intuition. "A space is a rather complex thing that is difficult to perceive globally." To study its structure, one may however project it on the real line. "In this flattening operation, the space resists: it reacts by creating singularities for the function. The singularities of the function are in some sense the vestiges of the topology that was killed: . . . its screams."[30] Publishing in 1955 his first article on singularities, Thom knew that he had found a great topic: "There is hardly any doubt . . . that the study of the local properties of singularities of differential mappings opens the door to an extremely rich domain."[31] His work on singularities provided him crucial mathematical tools for catastrophe theory: the concepts of *genericity* and of *structural stability*, as well as a classification of singularities later to become a list of the seven *elementary catastrophes*.

As an intuitive way of saying that some properties were much more common than others, the concept of genericity had been loosely used by Italian algebraic geometers since the beginning of the century. After a "memorable discussion" with the Bourbakist Claude Chevalley at Columbia in 1952, Thom had the idea of extending its use to other domains. "I quickly perceived that this phenomenon of 'genericity' was an essential source for our present worldview."[32] As for structural stability, it had been introduced from Russia (where it was known as roughness) by Princeton topologist Solomon Lefschetz, who since World War II had been reviving the qualitative study of ordinary differential equations and whom Thom visited in the early 1960s.[33] Structural stability was the assumption that, in order to be physically useful, systems had to exhibit similar behavior when slightly perturbed. That this concept was pivotal for catastrophe theory is reflected in the title of Thom's book, *Structural Stability and Morphogenesis*. Central to Stephen Smale's contemporary development of modern dynamical systems theory, the conjunction of genericity and structural stability likewise guided Thom's research program in singularity theory. Smale wished to show that structurally stable systems were generic; Thom, that structurally stable mappings were generic.[34]

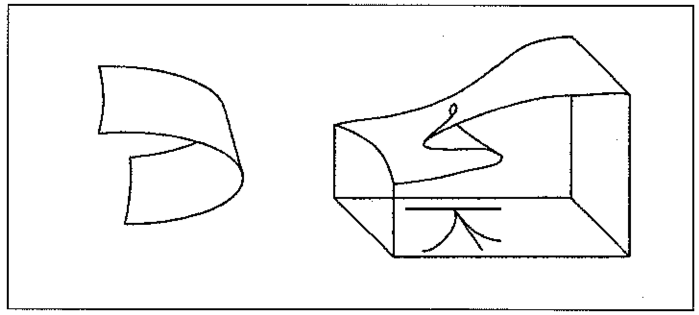
Mathematical interlude II: singularity theory

The "screaming" projection that René Thom described to show the importance of singularities was called a Morse function. It was a smooth mapping f from an n-dimensional manifold M to the real line $\mathbf{R}$. As Thom described it, one of Morse's crucial results allowed "the determination of the relations between the topological characteristics" of M and the singular points of f.[35] Consider a smooth differentiable mapping f from $\mathbf{R}^m$ to $\mathbf{R}^n$, or more generally from an m-dimensional manifold M to an n-dimensional manifold N. Then, a point p in M was a *singular point* of f if there was a direction along which the derivative of f at p vanished.

The name of the game then was, as often in modern mathematics, to classify and characterize singularities. For an arbitrary mapping f and arbitrary manifolds M and N, this was a very hard problem.[36] Thom focused on low-dimensional spaces and on structurally stable mappings, that is, those whose topological character was preserved under small perturbations. He hoped that structurally stable mappings would prove to be very common, so that every mapping was either stable or, in a topological sense, very close to one that was: in mathematical parlance, they were *generic*.

In the above example of real functions, a generic singular point p was such that the second derivative of f at p was nonzero: $f''(p) \neq 0$. Morse theory showed that using an appropriate change of variable $x \rightarrow y(x)$, such that $y(p) = 0$, then f could be written as $f(y) = \pm y^2$ in a small neighborhood. This completely classified the generic singular points for real functions: there was, essentially, only one kind of singularity that could occur, soon to be identified, in Thom's language, with the catastrophe called a *fold*.

Whitney completely classified the singularities that "a good approxima-



3. The fold and the cusp catastrophes.

tion" of any mapping from the plane to the plane was allowed to have.[37] He considered a surface S (a sheet, for example) projected on a plane underneath. The surface S was just a different parametrization of the plane. Often, there was no problem; there was a one-to-one correspondence between the points of S and those below. But it might happen that there was a fold, close to which two points from the surface were projected onto the same point in the plane; this was a singularity. Isolated points could even be encountered around which three points of S were projected onto the same point in the plane: these were *cusp* singularities. These two were the only local singularities that would survive small perturbations of the sheet.

Thom's elementary catastrophe theory basically extended this classification to higher dimensions, but with a slight difference. In *Structural Stability*, he recognized that the essential characteristics of a smooth function could be analyzed by studying its embedding into a smooth family of functions $F(x,u)$, such that $F(x,0) = f(x)$, which he called an *unfolding* of the function f (where x and u are multidimensional vectors). "The goal of catastrophe theory is to detect properties of a function by studying its unfoldings."[38] An infinite number of unfoldings existed for a given function f. The question was to know if one existed that captured the essential information about all of them. Such an unfolding, when it existed and the number of dimensions of the variable u was minimal, was called *universal*. The fold and the cusp, discussed earlier, were universal unfoldings of $f(x) = x^3$ and x^4, respectively (see figure 3). The tricky part of this program was to find universal unfoldings.

A beautiful, intriguing field of pure mathematics

The relationship between catastrophe theory and mathematics has always been contested. On the one hand, the mathematician John Guckenheimer wrote that *Structural Stability and Morphogenesis* "contains much of interest to mathematicians and has already had a significant impact upon mathematics, but [it] is not a work of mathematics." On the other hand, authors of recent textbooks often feel the need to stress its mathematical nature. One started by emphasizing that "catastrophe theory is a branch of mathematics." Another asserted that this branch had in fact been "discovered" by Whitney and transformed "into a 'cultural' tool" by Thom.[39]

There can be no doubt that Thom's mathematical experience made catastrophe theory possible and shaped his philosophy. As early as 1967, he divided catastrophes into two categories on the basis of his mathematical knowledge: the seven *elementary catastrophes* arising in simple systems and *generalized catastrophes*, which lived in more complex spaces arising with global loss of symmetry.[40] Thom wrote very little about the latter, since the mathematical basis for their classification was lacking. As for the former, they were those sudden discontinuities that occurred in systems whose dynamics were controlled by a gradient (or potential). The classic image "of a ball rolling around a landscape and 'seeking' through the agency of gravitation to settle in some position which, if not the lowest possible, then at least lower than any other nearby" was offered by Tim Poston and Ian Stewart.[41]

One of the most powerful results from singularity theory, and one that made catastrophe theory possible, was a complete classification of the elementary catastrophes that arose in systems described by less than four internal parameters. In this case, Thom conjectured that only seven elementary catastrophes existed: the *fold*, *cusp*, *swallowtail*, *butterfly*, and the three *umbilics*. Later widely known as "Thom's theorem," this conjecture was fully proved by Bernard Malgrange and John N. Mather, who used a heavy arsenal of functional analysis and algebraic topology.[42] Elementary catastrophe theory showed for certain that for gradient dynamical systems with a small number of parameters, abrupt generic changes had to be described locally by one of Thom's elementary catastrophes.

It was Christopher Zeeman's exploitation of Thom's theorem that made the international fame of catastrophe theory and later brought discredit to it. But this barely touched on Thom's own vision for his theory.[43] Too tight a focus on this theorem betrays his philosophy and misses the point of his most important innovations for the practice of modeling, a fact recognized by some catastrophists. "It is not Thom's *theorem*, but Thom's *theory*, that is the important thing: the assemblage of mathematical and physical ideas that lie behind the list of elementary catastrophes and make it work."[44]

Thom emphatically concurred with this view. He granted that advances in topology had made his philosophy possible and that mathematical concerns shaped his theory. Indeed, a larger body of qualitative mathematics

would have been quite beneficial for catastrophe theory. But such mathematical tools were just one facet of a general method of scientific inquiry.

Catastrophe theory is not a theory that is part of mathematics. It is a mathematical theory to the extent that it uses mathematical instruments for the interpretation of a certain number of experimental data. It is a hermeneutical theory, or even better, a methodology, more than a theory, aiming at interpreting experimental data and using mathematical instruments whose list is, for that matter, not a priori defined.[45]

The most casual reading of Thom's work reveals that his thought was framed by mathematical language. His emphasis on shapes and qualitative theories can be traced directly back to his work on topology, where measurements are eschewed, and on singularity theory, where global properties can be extracted from the local study of critical points. But Thom did not come up with catastrophe theory until he had experimented with biological theories. These are at least as important as his mathematical practice in explaining catastrophe theory. In fact, it was from his reading of embryology textbooks that he adopted the notion of *attractor*, later to figure prominently in the modeling and experimental practice of chaos.

Toward a theoretical biology?

Overlooking beautiful Lake Como, in the village of Bellagio, Italy, stands Villa Serbelloni owned by the Rockefeller Foundation. There, on 28 August 1966, a select group of computer scientists, mathematicians, physicists, and, of course, biologists (but hardly any molecular biologists!) gathered "to explore the possibility that the time [was] ripe to formulate some skeleton of concepts and methods around which Theoretical Biology [could] grow."[46] There also, René Thom introduced the notion of catastrophe. Far from being the first application of catastrophe theory to another discipline, Thom's theory of morphogenesis, as we shall see, grew out of his foray into embryology, which, at a mathematical level, helped him conceptualize the notion of attractor, and, at a philosophical level, gave Thom an example of a practice that used morphological raw materials.

From pure mathematics to theoretical biology, 1960–1968

In 1963, René Thom joined the faculty of the Institut des hautes études scientifiques (IHÉS), where he would have no teaching obligation and could devote most of his time to research, and where he would slowly move away from mathematics and venture into biology and linguistics.[47] At the IHÉS, he noted, "I had more leisure time, I was less preoccupied by teaching and administrative tasks. My purely mathematical productivity seemed to be declining and I began to be more interested in the periphery, that is, to possible applications." Perhaps he had finally succumbed to a taste for philosophy that he had neglected since his lycée years.[48] For all his success, Thom seemed to have found mathematics hard to practice and somewhat dissatisfying. "If you don't need to work in mathematics for a living you need much courage to do it, because, in spite of all, mathematics is difficult!"[49] However, he did not immediately abandon all concern with pure mathematics: throughout the 1960s he published articles on singularity theory, introducing many concepts picked up by other mathematicians.[50] Ultimately, one might concur with Zeeman: "In a sense Thom was forced to invent catastrophe theory in order to provide himself with a canvas large enough to display the diversity of his interest."[51]

In 1960, while in Strasbourg, Thom had already begun to experiment with caustics—those luminous outlines formed, for example, by sunlight in a cup of coffee. With singularities proving so fruitful in mathematics, he wondered whether they would be just as useful in the study of the physical world. Armed with a few simple instruments, he studied several caustics and their perturbations. The rays reflected by a spherical mirror, for example, formed a luminous curve with a cusp: a singularity. "This cusp has the marvelous property of being stable. If the orientation of the light rays is slightly changed, one sees that the cusp subsists. *This is the physical effect of a theorem of mathematics.*"[52]

Stumbling upon an unexpected behavior in optics, Thom then turned to biology. In 1961, while visiting the Natural History Museum in Bonn, he hit upon a plaster model of the gastrulation of a frog egg. "Looking at the circular groove taking shape and then closing up, I saw . . . the image of a cusp associated to a singularity. This sort of mathematical 'vision' was at the origin of the models I later proposed to embryology."[53] Thom also recalled that around 1962 he was struck by some mathematical models for biology: a proposal by the physicist Max Delbrück in 1949 to account for cell differentiation in terms of transitory perturbations of the cell's chemical environment, and Christopher Zeeman's articles on the topology of the brain, suggesting that topology could be applied to biological phenomena.[54]

In his preface to *Structural Stability and Morphogenesis*, Thom singled out four biologists as his precursors. In addition to D'Arcy Wentworth Thompson's classic *On Growth and Form*, he mentioned two "physiologists": Jakob von Uexküll and Kurt Goldstein.[55] In their works Thom found a way of treating organisms as wholes, a nonreductionist approach to biology that provided mechanisms accounting for the finality of living beings. Above all, he was impressed by the writings of the fourth man he cited: British biologist Conrad Hal Waddington. In 1968, Thom claimed two sources for his theory of morphogenesis: "On the one hand, there are my own re-

searches in differential topology and analysis on the problem called structural stability. . . . On the other hand, there are writings in Embryology, in particular those of C. H. Waddington whose ideas of 'chreod' and 'epigenetic landscape' seem to be precisely adapted to the abstract schema that I met in my theory of structural stability."[56]

This acknowledgment of Thom's—that his catastrophe theory derived also from embryology rather than having been merely applied to it—has rarely been taken seriously by commentators. But it is at the interface with biology that Thom would develop a mathematical picture of competition between attractors in dynamical systems—a picture that would become one of the cornerstones of both catastrophe and chaos theories.

"Wad" and the synthesis of biology

According to Waddington, the main problem of biology was to account for the characteristics that defined living organisms: form and end. "How does development produce entities which have Form, in the sense of integration or wholeness; how does evolution bring into being organisms which have Ends, in the sense of goal-seeking or directiveness?"[57] Organisms retained their shapes in spite of the fact that matter was continuously flowing through them. Development always ended up in the same final state, after having passed through the same stages. These problems of organization were fundamental questions, only to be solved by a synthesis of evolution, embryology, and genetics. Although Waddington believed that genes were the major cause for development, he never denied the influence of the rest of the organism. Thus, he thought that, while part of the answer lay in genetics, the main focus of study should not be the genes themselves but the nature of the causal relationship between the organism and its genes. For this science, he coined the name *epigenetics*.[58]

Being "stuck" with a biological order "in which there [was] an inescapable difference between the *genotype*—what is transmitted, the DNA—and the *phenotype*—what is produced when the genotype is used as instructions," the epigeneticist's task was to come up with mechanisms that could explain the phenotype in terms of the genotype.[59] Epigenetics had two main aspects: changes in cellular composition (cell differentiation), and changes in geometrical form (morphogenesis). Development followed definite pathways, which were resistant to change. The description of these pathways and the genetic influences on them was thus a major task of epigenetics. In 1939 Waddington introduced an intermediary space between the genotype and the phenotype, which he called the *epigenetic landscape*. In a unique visual representation, it combined all the development paths, which were pictured as valleys (figures 4 and 5).[60] The epigenetic landscape had no physical reality, but it helped visualize the various developmental processes.

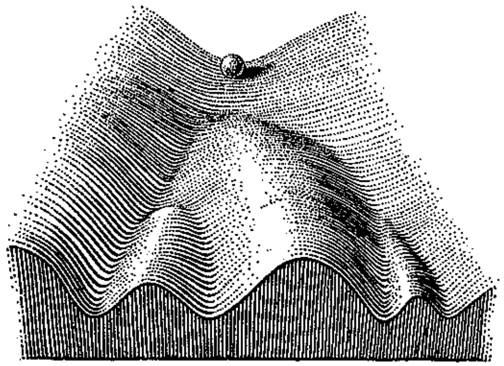
> Consider a more or less flat, or rather undulating surface, which is tilted so that points representing later states are lower than those representing earlier ones [figure 4]. Then if something, such as a ball, were placed on the surface it would run down towards some final end state at the bottom edge. . . . We can, very diagrammatically, mark along it one position to correspond, say, to one eye, and another to the brain.[61]

The image of the ball rolling down a surface is of course reminiscent of the potential functions of catastrophe theory. Moreover, the valleys formed on the epigenetic landscape had the property of being stable, in the sense that after a small perturbation in its trajectory, the ball tended to go back to the valley. These stable pathways of change, Waddington called *creodes*, and later *chreods*.[62] In his work on *Drosophila* during the 1930s, Waddington had studied the switches that can occur among several developmental paths. If a gene were active at a particular moment in the sequence of events, then the eye had a different tint of red. At the switches an important phenomenon took place. The ball had to choose among several pathways (figure 6). René Thom would see in this a topological change in the set of minima (singularities) of the potential function: a *catastrophe!*

Attractors in dynamical theories of morphogenesis

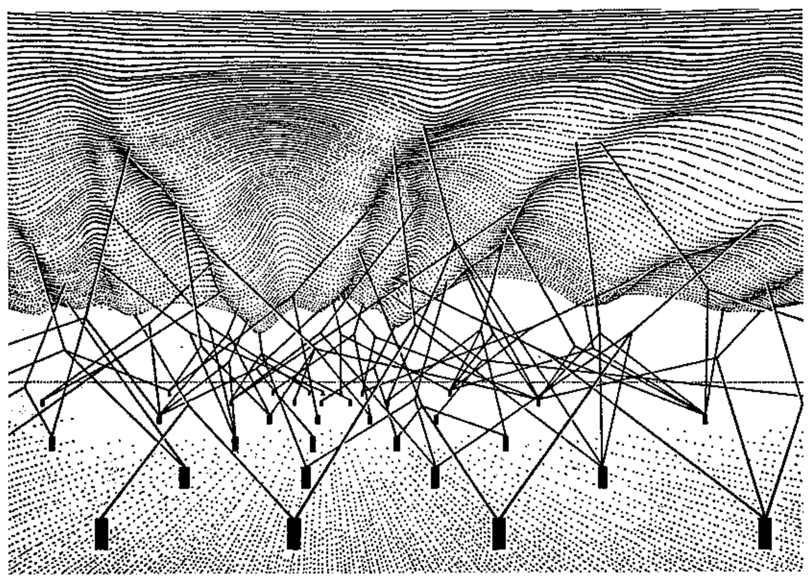
In his "dynamical theory of morphogenesis," Thom introduced a biochemical model of cellular differentiation. Independently, Waddington and Delbrück had proposed that gradients in the concentrations of some postulated chemical substance might account for the phenomenon.[63] In their schemes, the cell was constantly processing chemical substances so that the different concentrations changed in a complex way—given by coupled, nonlinear equations. In a biological system, a flux equilibrium was eventually reached; that is, concentrations remained stable even though chemical substances always flowed through the cell. Waddington and Delbrück considered that several stable regimes were achievable. The classification of these stable regimes became, in Thom's scheme, the description of the system's morphologies. Hence one of his most innovative ideas: to consider systems, even physical ones, in terms of the different end points they can reach, which he translated as a study of forms in nature. It expressed in a mathematical language adapted to the physical sciences the concept of finality in biology.

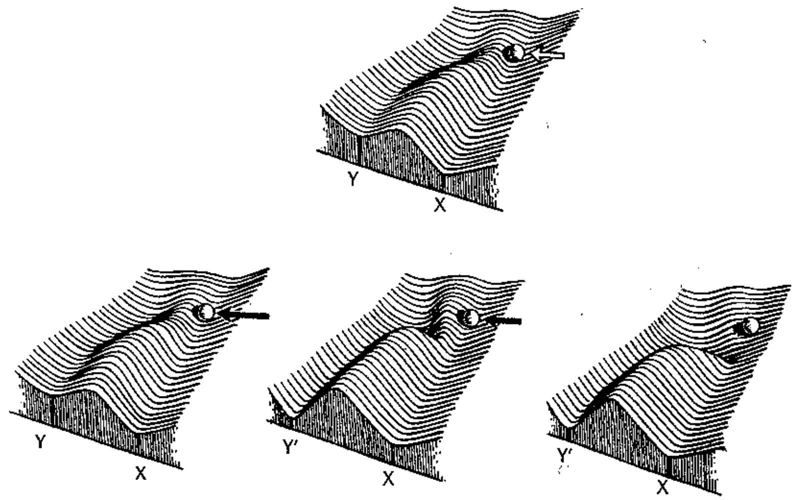
These different stable regimes of the system Thom called *attractors*.[64]



4. "*Part of an Epigenetic Landscape.* The path followed by the ball, as it rolls down towards the spectator, corresponds to the developmental history of a particular part of the egg. There is first an alternative, towards the right or the left. Along the former path, a second alternative is offered; along the path to the left, the main channel continues leftwards, but there is an alternative path which, however, can only be reached over a threshold." Source: Conrad Hal Waddington, *The Strategy of the Genes: A Discussion of Some Aspects of Theoretical Biology* (London: Allen and Unwin, 1957), 29.

5. "*The complex system of interactions underlying the epigenetic landscape.* The pegs in the ground represent genes; the strings leading from them the chemical tendencies which the genes produce. The modelling of the epigenetic landscape, which slopes down from above one's head towards the distance, is controlled by the pull of these numerous guy-ropes which are ultimately anchored to the genes." Source: Waddington, *Strategy of the Genes*, 36.





6. "*'Organic selection' (the Baldwin effect) and genetic assimilation.* The diagram above shows part of an epigenetic landscape, with a main valley leading to the adult character X and a side branch leading to Y; the developing tissue does not get into the Y path unless an environmental stimulus (hollow arrow) pushes it over the threshold. The three diagrams below show ways in which the 'acquired character' Y might become incorporated into the genotype. On the left, the original environmental stimulus is replaced by a mutant allele (dark arrow) which happens to turn up; this is 'organic selection.' On the right are two modes of 'genetic assimilation.' In the central one, the threshold protecting the wild type is lowered to some extent, but there is an identifiable major gene which helps push the developing tissues into the Y path. On the right, the genotype as a whole causes the threshold to disappear and there is no identifiable 'switch gene.' Note that in both the genetic assimilation diagrams there has been a 'tuning' of the acquired character, i.e., the Y valley is deepened and its end-point shifted from Y to Y′." Source: Waddington, *Strategy of the Genes*, 167.

They were regions of the configuration space stable under the dynamical equations of the system and such that any configuration close enough to an attractor would approach it asymptotically. The *basin of the attractor* was a region containing the attractor and inside of which any initial condition fell back to it. Of course Thom was aware that to achieve a complete topological description of attractors and basins of a general system would be a difficult task. It was an imaginable one, however, and, in essence, this task became a major focus for research on chaotic systems.

For local systems where, for example, the concentration of chemical substances was given at each point of space and time, attractors could differ from point to point. Thus the domain of space under study—the cell—was divided into several regions associated with different attractors.

These regions were separated by surfaces that Thom called "shock waves." Using Thom's theorem, he could establish that for gradient dynamics, these surfaces could only exhibit a small number of singularities, which were elementary catastrophes. Starting with a local singular situation in a dynamical system, he could say what ulterior catastrophes were contained in the "universal catastrophe space" associated with the singularity. For example, if one started with a local critical cusp situation, the only other catastrophes that could occur later were folds. Of course, all of this was local in a topological sense: some finite time limit existed beyond which anything could happen. There was no way of knowing how large this limit was; it could be as small as one wished but not zero. It could even be impossible to detect; hence Thom was reluctant to accept that catastrophe theory could be submitted to experimental control.

In his theory, Thom saw "a mathematical justification for the idea of 'epigenetic landscape,' suggested 20 years earlier by Waddington."[65] This was not mere gesture: the ideas of conflicting attractors had been described almost word for word by the biologist.

> [1] At each step [of development] there are several genes acting, and the actual development which occurs is the result of a balance between opposing gene-instigated tendencies. [2] At certain stages in the development of an organ, the system is in a more than usually unstable condition, and the slightest disturbances at such times may produce large effects on later events. . . . [3] An organ or tissue is formed by a sequence of changes which can be called the "epigenetic paths." . . . And also each path is "canalized," or protected by threshold reactions so that if the development is mildly disturbed it nevertheless tends to regulate back to the normal end-result.[66]

Although he hardly knew enough mathematics, Waddington claimed that Thom had "shown how such ideas as chreods, the epigenetic landscape, switching points, etc.,—which previously were expressed only in the *unsophisticated language of biology*—can be formulated more adequately."[67] However, catastrophe theory explained biological structures by describing "the basic and universal constraints of stability imposed on epigenetic mechanisms," independently of DNA, and therefore never answered the question that had prompted Waddington to imagine epigenetic landscapes and chreods in the first place, that is, the link between development and genetics. Contentiously, Thom insisted that "only a mathematician, a topologist, could have written [this theory], and the time may be very near when, even in biology, it might be necessary to think."[68]

Revealing contrasts exist between Thom's writings on biology and those of Jacques Monod, the Nobel prize-winning molecular biologist whose work would reach a broad audience. A chapter of Monod's *Chance and Necessity*, published in 1970, was devoted to the problem of spontaneous morphogenesis of living organisms. But Monod's picture was almost totally opposed to Thom's. Indeed, Monod explained his aims as follows:

> In this chapter I wish to show that this process of spontaneous and autonomous *morphogenesis* rests, at bottom, upon the stereospecific recognition properties of proteins; that is *primarily a microscopic process* before manifesting itself in macroscopic structures. . . . But we must hasten to say that this "reduction to the microscopic" of morphogenetic phenomena does not yet constitute a working theory of phenomena. Rather, it simply sets forth the principle in whose terms such a theory would have to be formulated if it were to aspire to anything better than simple phenomenological description.[69]

As opposed to Thom's reduction of morphogenetic processes to a certain mathematical idealism, Monod argued for the "principle" of reducing them to molecular interaction. As Monod's remarks indicate, this was nothing more than a "principle" and certainly not a full theory. But Monod put a great deal of faith in this principle.

> I for my part remain convinced that only the shape-recognizing and stereospecific binding properties of proteins will in the end provide the key to these [morphogenetic] phenomena. . . . In a sense, a very real sense, it is at the level of chemical organization that the secret of life lies, if indeed there is any one such secret.[70]

Emphasizing the molecular and chemical properties of the substratum, the forces acting between organic macromolecules, and quantitative studies, Monod's discourse sounded like a diatribe directed at Thom, or conversely.[71] Just as uncompromising, the mathematician emphasized that no theoretical explanation was conceivable in biology without the aid of mathematics.

> There should not exist any other theorization than mathematical; concepts used in each discipline that are not susceptible of gathering a consensus around their use (let us think, for example, of the concept of information in Biology) should be progressively eliminated after having fulfilled their heuristic function. In this view of science, only the mathematician, who knows how to characterize and generate stable forms in the long term, has the right to use (mathematical) concepts; *only he, at bottom, has the right to be intelligent.*[72]

In this context, one is hardly surprised by the fact that Thom's theory had little impact on biology.[73] However, his forays into embryology provided Thom with crucial intuition about ways to study dynamical systems with finality. In no small sense, his introduction of the concept of the attractor, and the even more important concept of the basin of an attractor, can be seen as stemming from his involvement in biology.

Topology and meaning

Having pointed out the relevance of topological concepts and practices for the modeling of biological phenomena, Thom saw no reason to stop there. Since the early 1970s, his main fields of research, besides philosophy, have been linguistics and semiotics. With his incursion into the human sciences, Thom was bound to confront structuralism. Never himself a structuralist per se, but trained in the mathematical structuralism of Bourbaki, he was attracted by this movement. With some adjustments, his theories could be made to fit into structuralist modes of thought. But because he began to work on linguistics so late, catastrophe theory was only mildly affected by structuralism in practice. Increasingly faced with strong opposition to his ideas about modeling, Thom pondered the epistemological foundations of catastrophe theory. In attempting to articulate the kind of knowledge that the theory produced, he used structuralist resources most obviously.

Catastrophes, man, and language

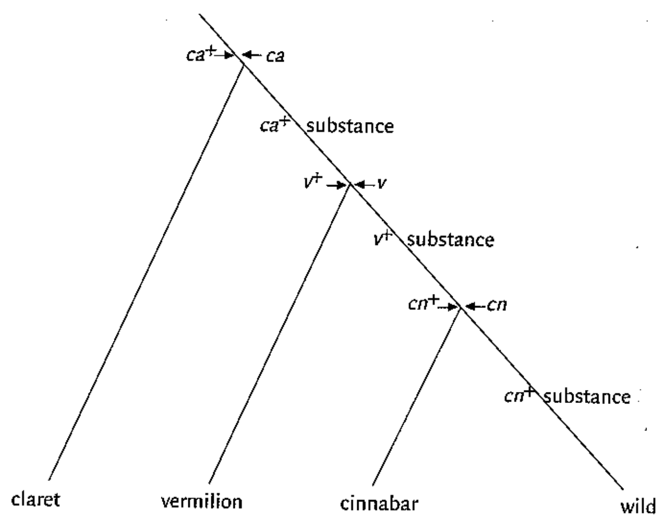
In his manuscript of *Structural Stability and Morphogenesis*, Thom titles chapter 13 "L'homme." It would be published with substantial additions under the title "From Catastrophes to Archetypes: Thought and Language." The original chapter aimed at extending the techniques and assumptions of catastrophic models of morphogenesis to human thought processes and societies. He actually developed few of the models he suggested. Always a mathematical terrorist, Thom used mathematical notations and language only to express vague correspondences among neurological states, thoughts, and language.

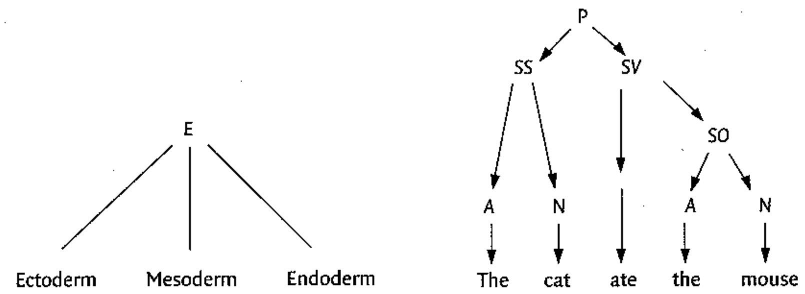
His basic assumption was that there existed a few "functional chreods," later renamed "archetypal chreods," that expressed simple biological actions: to throw a projectile, to capture something, to reproduce, and so forth. These chreods had been internalized in the human brain, whose mental activity (*activité psychique*) he identified with a dynamical system. By analogy with the epigenetic landscape, Thom postulated that this psychological system was divided among basins and attractors. "The sequence of our thoughts and our acts is a sequence of attractors, which succeed each other in 'catastrophes.' "[74]

Language, Thom then claimed, was a translation of these mental attractors. A mental atlas of dynamic chreods existed that was common to all human beings. An idea was a mental attractor. When one wished to formulate a sentence expressing an idea, it was mathematically projected onto a space of admissible sentences, where several attractors competed. One was eventually chosen, and the sentence was uttered. All this was manifestly programmatic and rather vague.

In the Parisian intellectual climate of the late 1960s, Thom could not avoid structuralism, especially since he dealt with language. As early as 1968, he noted that "the problem of meaning has returned to the forefront of philosophical inquiry."[75] Nevertheless, semiotics was first introduced in Thom's work not as a quest in itself, but as a method for biology. Indeed, he considered Saussure's notions of *signified* and *signifier* as congenial to the goals of epigenetics, which were to find the connections between genetics and embryology. "Is not such a discipline which tries to specify the connection between a global dynamic situation [the organism] (the 'signified'), and the local morphology in which it appears [DNA] (the 'signifier'), precisely a 'semiology'?" He portrayed his method for morphogenesis as a problem of semantics. "The decomposition of a morphological process taking place in R^m can be considered as *a kind of generalized m-dimensional language*; I propose to call it a *'semantic model.'* "[76]

7. Waddington's switching diagram. "The formation of eye colors in *Drosophila*. The pigment-forming process normally runs down the line through the ca^+ substance, the v^+ substance, and the cn^+ substance, to give wild type pigment. The genes, *ca*, *v* and *cn* interrupt this sequence, so that the process takes an altered course, to give claret, vermilion or cinnabar pigmentation." Source: Conrad Hal Waddington, *Organisers and Genes* (Cambridge: Cambridge University Press, 1940), 77.





8. Thom's analogy between graphs of sentences and development. Source: Thom, "Structuralism and Biology," in Conrad Hal Waddington (ed.), *Towards a Theoretical Biology* (Edinburgh: Edinburgh University Press, 1972), 4:80.

In 1970, Thom presented a more sophisticated catastrophe-theoretical model of language.[77] His goal was to explain the syntactical structure of atomic sentences (basically, those with one verb), in terms of their meaning. He was struck by the resemblance between the tree-shaped graphs that linguist Louis Tesnière used to analyze the structure of sentences and Waddington's chreods (figures 7 and 8).[78] Indeed, stripped of the out-of-equilibrium position, the epigenetic landscape became a switching diagram, like a tree. In Tesnière's view, verbs were the center of gravity of sentences. They became, in Thom's view, the attractors of mental activities, and words were chreods. He developed a visual representation of the verbs associated with spatiotemporal activities by using sections of elementary catastrophe surfaces. This was, he would say twenty years later, a "geometrization of thought and linguistic activities."[79] The main benefit of such an analysis was to establish a map from signified to signifier, which went against the Saussurean dogma that the relation between signifier and signified is arbitrary. Classifying syntactical structures into sixteen categories, Thom claimed that "the topological type of the interaction determines the syntactical structure of the sentence which describes it."[80] Meaning and structure were no longer independent. Thus, with catastrophe theory and the biological analogy, Thom subverted classical structuralist ideas, which explains why his work would be picked up by philosophers like Michel Serres and Jean-François Lyotard, who explicitly opposed the structuralist project.[81]

Structuralism and biology: explaining forms

Thom confronted structuralism head on in 1972: "Can structuralist developments in anthropological sciences (such as linguistics, ethnology, and so on) have a bearing on the methodology of biology? I believe this is so." By then what he called structuralism was merging with his own method.

> The task of any structuralist theory is: (1) to form a finite lexicon of elementary chreods; (2) to build experimentally the "corpus" of the empirical morphology; . . . (3) to define "conditional chreods," objects of the theory; (4) to describe the internal structure of a conditional (or elementary) chreod by associating a mathematical object to it, whose internal structure is isomorphic to the structure of the chreod.[82]

Catastrophe theory and structuralism reinforced each other. The semantic analogy was common among molecular biologists who were prone to interpret the living order in terms of the DNA code as a static linear string. For Thom, this was wrongheaded because if biology could indeed be seen as a semantic model, it was a dynamical, multidimensional one. The DNA code by itself was a semantic model of dimension one: how could it describe the spatial processes of biology?[83]

In contact with structuralist linguistics, Thom extracted a philosophy of science that he thought would be able to make sense of the knowledge his approach had produced, and not only in the human sciences. Henceforth, Thom distinguished two approaches to scientific knowledge: the *reductionist* and the *structural*.[84] Both approaches aimed at simplifying the description of empirically observed morphologies. But the latter refused to do so by attributing causal effects to factors outside the observed morphology. The only admissible causality was structural.

Thom had obviously modeled his "structural approach" on the linguists' claims to knowledge production. He thought some human sciences had succeeded in building nonreductionist theories, especially formal linguistics and Lévi-Strauss's structural analysis of myths. They held a "paradigmatic value: they show the way in which a purely structural, morphological analysis of empirical data can be engaged." It would indeed be absurd, Thom contended following Lévi-Strauss, to base linguistics on reductionist assumptions. "It would consist in an attempt at explaining the syntactical structure of a sentence of words by an interaction of phonemes of a phonologic character."[85]

Like Swiss psychologist Jean Piaget, Thom saw a serious epistemological problem in structuralism, namely that it could not account for the emergence of its structures because, historically, structuralist linguistics was synchronic, that is, static in time.[86] Thus the hope for the future lay in synthesizing both approaches. Nothing prevented linguists from conceiving time as another dimension of space-time: "we can make a structural

theory of the changes of forms, considered as a morphology on the product space of the [conjoined] substrate space and time axis."[87] Indeed, catastrophe theory provided a way of building a dynamic structuralism that would explain the emergence of structure. As he had done with the structuralist mathematics of Bourbaki, Thom used structuralist linguistic practices to undermine the very project of structuralism.

Shapes, *logoi*, and catastrophes: Thom's philosophy of science

As we have seen, René Thom was among those who loudly contested the success of reductionist science. That science in the twentieth century had been mainly a reductionist enterprise was a commonplace. In their efforts to understand the world—or, more precisely, pursuing the Laplacian dream, to predict its future course—scientists followed Jean Perrin's ideal: "to explain complex visible things with the help of simple invisible things."[88] Thom contended that this approach was far from having lived up to its promises. "The Universe is nothing more than a brew of electrons, protons, [and] photons," he wrote. "How can this brew settle down, on our scale, into a relatively stable and coherent form far from the quantum-mechanistic chaos?"[89]

Thom believed that physicists overreached themselves when they claimed to be able to explain the everyday world. "Realization of the ancient dream of the atomist—to reconstruct the universe and all its properties in one theory of combinations of elementary particles and their interactions—has scarcely been started." Thom adamantly opposed dogmatic reductionism: "this primitive and almost cannibalistic delusion about knowledge, [which demands] that an understanding of something requires first that we dismantle it, like a child who pulls a watch to pieces and spreads out the wheels in order to understand the mechanism."[90] He did recognize reductionism, in principle, as a valid approach to knowledge, but one which was unachievable at the practical level.

"Reality presents itself to us as phenomena and shapes."[91] Thom's program was to make the morphologies of our day-to-day reality the object of a dynamical science of shapes. In a given domain of experience, his modeling practice could be summarized as follows: find the shapes that are usually encountered, establish a list of these shapes according to their topologic character, and find the underlying dynamics that governs their emergence and destruction.[92] Thom took his cue from D'Arcy Thompson, who had recognized the morphological problems arising in the physical sciences. Thompson, however, had confidence that physics was capable of explaining morphologies. "The waves of the sea, the little ripples on the shore, the sweeping curve of the sandy bay between the headlands, the outline of the hill, the shape of the clouds, all these are so many riddles of form, so many problems of morphology, *and all of them the physicist can more or less easily read and adequately solve*."[93] Listing similar natural shapes, Thom disagreed that traditional physics could do it:

> Many phenomena of common experience, in themselves trivial (often to the point that they escape attention altogether!)—for example, the cracks in an old wall, the shape of a cloud, the path of a falling leaf, or the froth on a pint of beer—are very difficult to formalize, but is it not possible that a mathematical theory launched for such homely phenomena might, in the end, be more profitable for science [than large particle accelerators]?[94]

Catastrophe theory was thus an attempt at formalizing in rigorous mathematical language a dynamics of forms. And in *Structural Stability*, the first seven chapters gave an outline of a general theory of morphology, applicable to all problems of shape.

But it was one thing to focus on forms and quite another to focus on the specific ones that he and Thompson listed. Just as Thom questioned the pertinence to the everyday world of explanations in terms of electrons, he also noticed that science was quite unable to account for "the froth on a pint of beer." The French mathematician Benoît Mandelbrot, inventor of fractals, shared this concern. "Clouds are not spheres, mountains are not cones."[95]

If Mandelbrot saw himself as a new Euclid, Thom thought of himself as picking up a broken line of thought just where Heraclitus had left it. Around 500 BC, Greek philosopher Heraclitus already noticed the difference between knowledge and understanding. "Many people do not understand the sorts of things they encounter! Nor do they recognize them even after they have had experience of them, though they themselves think [they do]."[96] In Heraclitus's fragments, Thom found inspiration. Christening his elementary catastrophes with names like *swallowtail* and *butterfly*, he applied Heraclitus's precept to irregular figures impossible to visualize.[97]

Once a description of natural forms was achieved, the next pressing concern was their stability, especially if one believed that they emerged from a "brew of electrons." Returning to his quarrel with reductionist physics, Thom noticed that "although certain physicists maintain that the order of our world is the inescapable consequence of elementary disorder, they are still far from being able to furnish us with a satisfactory explanation of the stability of common objects and their qualitative properties." In other words, the physicists were not able to understand the morphologies of the world in terms of atoms.

Thom believed that the explanation lay in an ideal mathematical structure. "The stability of a form rests definitively upon a structure of algebraic-geometric character . . . endowed with the property of *structural stability* with respect to the incessant perturbations affecting it. It is this algebraic-geometric entity that I propose, recalling Heraclitus, to call the *logos* of the form."[98] For Heraclitus, the *logos* was the "true discourse according to which everything happens. It was the truth of this world."[99] Thom attributed a logos to each form; it was "a formal structure which insures its unity and stability." One may note here that he was indeed applying Perrin's precept, except that Thom's "simple invisible things" were mathematical structures as opposed to atoms. For all his structuralist talk, Thom's philosophy is well captured by the term "neoreductionism," a word that Giorgio Israel has used to characterize von Neumann's approach.[100]

Thom felt he needed to emphasize that he studied morphology without regard to the substrate. In his manuscript, written in 1966, he had not mentioned this.[101] As a Bourbakist topologist, he believed in the universal relevance of his mathematics. But after having presented his theory to biologists, he underscored its independence from specific material bases. "The essence of our theory, which is a certain knowledge of the properties peculiar to the substrates of the forms, or the nature of the forces at work, may seem difficult to accept, especially on the part of experimenters."[102]

Again, Thom saw himself as heir to D'Arcy Thompson, who, "in some pages of rare insight, compared the form of a jellyfish to that of the diffusion of a drop of ink in water."[103] The only thing that Thompson lacked, Thom added, was a formal foundation in topology, which provided the basis for explaining morphogenesis without relying on material properties. He endorsed a strong idealism: "The hypothesis that Platonic ideas give shape to the universe," he wrote in 1970, "is the most natural and, philosophically, the most economical."[104]

There was a drawback to this all-encompassing vision. Based on topology, Thom's method was not suited to quantitative analysis and measurement. It had to remain qualitative. Traditionally, this was a serious problem: "probably [one of scientists'] most deeply held values concerns predictions; . . . quantitative predictions are preferable to qualitative ones."[105] But Thom saw the qualitative aspect of catastrophe theory in a positive light. To understand the world one had to rid oneself of "the intolerant view of dogmatic quantitative science." To expose this common prejudice, he loved to recall Rutherford's dictum: "Qualitative is nothing but poor quantitative!" But "what condemns these speculative [qualitative] theories in our eyes," Thom wrote, "is not their qualitative character but the relentlessly naive form of, and the lack of precision in, the ideas they use." Now, he claimed, everything had changed since one could "present qualitative results in a rigorous way, thanks to recent progress in topology and differential analysis, for we know how to define a *form*."[106] Catastrophe theory was the rigorous way to think about quality.

Intelligibility of the world was the benefit and the ultimate goal of Thom's approach. He defended Descartes against Newton: "Descartes, with his vortices, his hooked atoms, and the like, explained everything and calculated nothing; Newton, with the inverse square law of gravitation, calculated everything and explained nothing." Again and again, Thom opposed explanation to prediction, intelligibility to control, understanding to action: "just as precise knowledge of a pathology often makes us anticipate, powerlessly, the sickness and death of a dear one, it is not impossible that an increased understanding will make us foresee the development of a catastrophe, a catastrophe whose theory will make us know the very reason of our powerlessness."[107] Ultimately, he believed that a theory would be totally intelligible when the theory itself could decide on its own validity: "*a theory of meaning whose nature is such that the act of knowing itself is a consequence of the theory*."[108] While Thom never claimed that catastrophe theory could live up to this feat, he nevertheless thought that it made the world more intelligible.

He often insisted that catastrophe theory was not a proper scientific theory. It was a language, a method. Nowhere was this more evident than when he confronted the delicate question of experimental control. He always admitted that an experiment that would falsify or, for that matter, confirm his theories was in principle impossible because catastrophe theory was inherently qualitative. It might eventually provide the basis for elaborating a quantitative model susceptible of experimental control, but in general, the necessary mathematics did not yet exist. And even if it were possible to analyze mathematically the dynamical processes that insured the stability of a form, "this analysis is often arbitrary; it often leads to several models between which we can only choose for reasons of economy or mathematical elegance."[109]

But, once again, according to Thom, the drawback was not fatal. He saw at least two reasons to justify scientists' interest in the theory. First, catastrophe theory questioned the traditional "qualitative carving out of reality . . . into the big disciplines: Physics, Chemistry, Biology."[110] It would integrate this taxonomy of experience into "an abstract general theory, rather than blindly accept[ing] it as an irreducible fact of reality." Second, catastrophe theory could replace the "lucky guess" of previous model construction in science. "The ultimate aim of science is not to amass undifferentiated empirical data," he wrote, "but to organise this data in a more or less formalised structure, which subsumes and explains it."[111] On the path toward a "general theory of models," catastrophe theory showed the way of the future.

Conclusion

With catastrophe theory, René Thom believed that he was breaking away from centuries of reductionist thinking. He developed models for biology, linguistics, and semiotics displaying his vision of a structural science. He introduced a new modeling practice and tried to codify its epistemological rules. Based on his mathematical experience, catastrophe theory used topology as a resource for grasping a world of qualities and shapes. Embryology suggested to him a new starting point for theory, namely the end of a dynamic process: its morphology. Thom never argued for the intrinsic superiority of his method, but rather for its greater capacity to explain the world as it is perceived. Catastrophe theory provided "schemes of intelligibility. And this seems quite valuable to me."[112]

In developing catastrophe theory, Thom introduced important mathematical concepts and attempted to extend them beyond their rigorous limits. In doing so, his speculations were often rejected by mathematical communities. His insistence on denying the possibility of experimentation was met with suspicion by practicing biologists. Finally, it was the nongenericity of structural stability for nongradient systems that discredited the general ambitions of catastrophe theory. As for elementary catastrophe theory applied to the physical sciences, it did not seem to explain anything that was not already known.

Thom's program, however, was richer than just concepts, models, theorems, and theories. His modeling practice presented some appealing aspects, some explanatory strategies that would be taken up by "chaologists." In 1971, using Thom's concept of attractors and his geometric vision of dynamical systems, David Ruelle and Floris Takens conjectured that the attractor usually posited for turbulence was not structurally stable, and thus introduced the notion of *strange attractors* at the roots of chaos theory.[113] But, contrary to Thom's model, theirs was successfully submitted to the verdict of experiments, in the laboratory and on the computer. This would be a decisive difference.

Notes

For their help and comments I am indebted to the participants in the Princeton workshop, to the graduate students of Princeton University's program in the history of science, and to the members of my doctoral committee, M. Norton Wise, Amy Dahan Dalmedico, and Michael S. Mahoney. I thank the Institut des hautes études scientifiques of Bures-sur-Yvette and its director Jean-Pierre Bourguignon for generous access to their archives. I am grateful to René Thom for having commented on an earlier version and responding to my questions. This work was supported by the Social Science and Humanities Research Council of Canada and the John C. Slater Fellowship of the American Philosophical Society.

1 René Thom, "La science malgré tout . . . ," *Encyclopædia Universalis*, vol. 17: *Organum* (1975), 5, 7. All translations are mine except when I quote from a published translation. For a French view of the crises facing science in the 1970s, see Alain Jaubert and Jean-Marc Lévy-Leblond (eds.), *(Auto)critique de la science* (Paris: Seuil, 1975).

2 Thom, "La science malgré tout . . . ," 10; my emphasis. Thom mentions the "prime vocation" of science on p. 5.

3 Only accounts written by scientists exist concerning the history of catastrophe theory. See Alexander Woodcock and Monte Davis, *Catastrophe Theory* (New York: E. P. Dutton, 1978), which is also a good nontechnical introduction to the subject; Ivar Ekeland, *Le Calcul, l'imprévu: Les figures du temps de Képler à Thom* (Paris: Seuil, 1984), translated as *Mathematics and the Unexpected* (Chicago: University of Chicago Press, 1988); Tito Tonietti, *Catastrofi: Una controversia scientifica* (Bari: Dedalo, 1983); and Vladimir I. Arnol'd *Catastrophe Theory*, 3d rev. and expanded ed., trans. G. S. Wasserman, based on a translation by R. K. Thomas (Berlin: Springer, 1992); see also John Guckenheimer, "The Catastrophe Controversy," *Mathematical Intelligencer* 1 (1978): 15–20; and Alain Boutot, "Catastrophe Theory and Its Critics," *Synthese* 96 (1993): 167–200.

4 Domenico P. L. Castrigiano and Sandra A. Hayes, *Catastrophe Theory* (Reading: Addison-Wesley, 1993), xii; see also Thom's assessment of this in "René Thom répond à Lévy-Leblond sur la théorie des catastrophes," *Critique* 33(361/362) (1977): 681.

5 René Thom, *Prédire n'est pas expliquer*, interview by Émile Noël (Paris: Eshel, 1991), 47; my emphasis.

6 David Aubin, "Chaos et déterminisme," in Dominique Lecourt (ed.), *Dictionnaire d'histoire et de philosophie des sciences* (Paris: Presses universitaires de France, 2000), 166–168.

7 The notion that most domains of science have emphasized shapes and forms in their development during the last decades has been shown in a recent special issue of *La Recherche*, no. 305 (January 1998), devoted to the "origin of forms."

8 René Thom, "La linguistique, discipline morphologique exemplaire," *Critique* 30 (1974): 245.

9 On the link between mathematics and the social sciences in France at an earlier period and the manner in which catastrophe theory intervened in this context, see David Aubin, "The Withering Immortality of Nicolas Bourbaki: A Cultural Connector at the Confluence of Mathematics, Structuralism, and the Oulipo in France," *Science in Context* 10(2) (1997): 297–342.

10 For a historiographical review of chaos, see David Aubin and Amy Dahan Dalmedico, "Writing the History of Dynamical Systems and Chaos: *Longue Durée* and Revolution, Discipline and Culture," *Historia Mathematica* 29 (2002): 273–339. The standard account of chaos as scientific revolution is James Gleick, *Chaos: Making a New Science* (New York: Viking, 1987). The philosopher Stephen Kellert speaks of "dynamic understanding" in his book *In the Wake of Chaos: Unpredictable Order in Dynamical Systems* (Chicago: University of Chicago Press, 1993).

11 See David Aubin, "A Cultural History of Catastrophes and Chaos: Around the Institut des Hautes Études Scientifiques, France" (Ph.D. diss., Princeton University, 1998); and "From Catastrophe to Chaos: The Modeling Practices of Applied Topologists," in U. Bottazzini and A. Dahan Dalmedico (eds.), *Changing Images in Mathematics: From the French Revolution to the New Millennium* (London: Routledge, 2001), 255–279.

12 René Thom, *Stabilité structurelle et morphogénèse* (Reading, Mass.: W. A. Benjamin, 1972; Paris: InterÉditions, 1977); *Structural Stability and Morphogenesis*, trans. David H. Fowler (Reading, Mass.: Benjamin, 1975); hereafter cited as *SSM*.

13 René Thom, "Une théorie dynamique de la morphogénèse," in Conrad Hal Waddington (ed.), *Towards a Theoretical Biology*, vol. 1 (Edinburgh: Edinburgh University Press, 1968); and "Topologie et signification," in *L'Âge de la science* 4 (1968). Both of these articles are reprinted in René Thom, *Modèles mathématiques de la morphogénèse: Recueil de textes sur la théorie des catastrophe et ses applications* (Paris: Union générale d'éditions, 1974; Paris: C. Bourgois 1980); *Mathematical Models of Morphogenesis*, trans. W. M. Brookes and D. Rand (Chichester: Ellis Horwood, 1983), 1–38, 166–191; hereafter cited as *MMM*.

14 See, e.g., René Thom, "Towards a Revival of Natural Philosophy," in W. Güttinger and H. Eikemeier (eds.), *Structural Stability in Physics* (Berlin: Springer, 1979), 5–11. See also Jean Largeault, "René Thom et la philosophie de la nature," *Critique* 36 (1980): 1055–1060.

15 Thom, "Une théorie dynamique," *MMM*, 15.

16 René Thom, "Exposé introductive," in Jean Petitot (ed.), *Logos et Théorie des catastrophes: À partir de l'oeuvre de René Thom*, Actes du colloque international de Cerisy-la-Salle, 1982 (Geneva: Patiño, 1988), 24, where he also wrote that "sociologically" (*sociologiquement*), he was a mathematician. Thom recounts his memories in two published interviews: *Paraboles et catastrophes: Entretiens sur les mathématiques, la sciences et la philosophie*, interview by G. Giorello and S. Morini (Paris: Flammarion, 1983) and *Prédire n'est pas expliquer*. See also "Problèmes rencontrés dans mon parcours mathématique: un bilan," *Publications mathématiques de l'IHÉS* 70 (1989): 199–214, and his interview in Marion Schmidt (ed.), *Hommes de sciences: 28 portraits* (Paris: Hermann, 1990), 228–234.

17 Thom, "Problèmes rencontrés," 200.

18 Liliane Beaulieu, "Bourbaki. Une histoire du groupe de mathématiciens français et de ses travaux (1934–1944)," thèse de l'université de Montréal (1989), 1; see also Aubin, "The Withering Immortality."

19 David Hilbert and S. Cohn-Vossen, *Geometry and the Imagination*, trans. P. Nemenyi (1932; New York: Chelsea, 1952), iii; their emphasis.

20 Interviewer's comment, to which Thom agrees, in *Paraboles et catastrophes*, 24.

21 Thom, *Paraboles et catastrophes*, 23; see also André Haefliger, "Un aperçu de l'œuvre de Thom en topologie différentielle (jusqu'en 1957)," *Publications mathématiques de l'IHÉS* 68 (1988): 15.

22 Jean Dieudonné, *Panorama des mathématiques pures: Le choix bourbachique* (Paris: Gauthier-Villars, 1977), 14; see also Thom's paper "Quelques propriétés globales des variétés différentiables," *Commentarii Mathematici Helvetici* 28 (1954): 17–86.

23 Heinz Hopf, "The Work of R. Thom," in *Proceedings of the International Congress of Mathematicians*, Edinburgh, August 1958 (Cambridge: Cambridge University Press, 1960), lxiii–lxiv.

24 Tim Poston, quoted by Woodcock and Davis, *Catastrophe Theory*, 16.

25 René Thom, "Mathématiques modernes et mathématique de toujours," in Robert Jaulin (ed.), *Pourquoi la mathématique?* (Paris: Union générale d'éditions, 1974), 49; and in *Entretiens avec "Le Monde,"* interview by Jean Mandelbaum (Paris: La Découverte, 1984), 3:52, 80; see also René Thom, "Modern Mathematics: An Educational or Philosophical Error?" *American Scientist* 51 (1971): 697; repr. in *New Directions in the Philosophy of Mathematics* (Boston: Birkhäuser, 1986), 67–78; originally published in French in *L'Âge de la science* 3(3) (1970): 225–236; repr. in *Pourquoi la mathématique?*, 57–88.

26 Thom, "Quelques propriétés globales"; see also "Sous-variétés et classes d'homologie des variétés différentiables," *Séminaire Bourbaki* 5 (February 1953), exposé #78; and "Variétés différentiables cobordantes," *Comptes-rendus de l'Académie des Sciences* 236 (1954): 1733–1735.

27 Thom, "Exposé introductif," 27; my emphasis.

28 René Thom, "La vie et l'œuvre de Hassler Whitney," *Comptes-rendus de l'Académie des sciences–La vie des Sciences* 7 (1990): 473–476.

29 Paul Montel, "Sur les méthodes récentes pour l'étude des singularités des fonctions analytiques," *Bulletin des sciences mathématiques* 2d ser., 56 (1932): 219.

30 Thom, "Exposé introductif," 26.

31 René Thom, "Les singularités des applications différentiables," *Annales de l'Institut Fourier de Grenoble* 6 (1955–56): 87; see also "Les singularités des applications différentiables," *Séminaire Bourbaki* 8 (May 1956), exposé #134; and see Bernard Teissier, "Travaux de Thom sur les singularités," *Publications mathématiques de l'IHÉS* 68 (1988): 19–25; Haefliger, "Un aperçu," 16.

32 René Thom, "Mémoire de la théorie des catastrophes," in R. Thom, M. Porte, and D. Bennequin (eds.), *La genèse de formes*. I thank R. Thom and M. Porte for providing me a copy of this text.

33 Thom, "Exposé introductif," 31. On Lefschetz, see Amy Dahan Dalmedico, "La renaissance des systèmes dynamiques aux États-Unis après la deuxième guerre mondiale: l'action de Solomon Lefschetz," *Rendiconti dei circolo matematico di Palermo*, ser. II, Supplemento, 34 (1994): 133–166. On the Russian school, see Simon Diner, "Les voies du chaos déterministe dans l'école russe," in Amy Dahan Dalmedico, Karine Chemla, and Jean-Luc Chabert (eds.), *Chaos et déterminisme* (Paris: Seuil, 1992), 331–370; see also Aubin and Dahan Dalmedico, "Writing the History of Dynamical Systems" and references therein.

34 Aubin, "From Catastrophe to Chaos." About Smale, see Stephen H. Smale, *The Mathematics of Time: Essays on Dynamical Systems, Economic Processes, and Related Topics* (New York: Springer, 1980); Morris W. Hirsch, Jerrold E. Marsden, and Michael Shub (eds.), *From Topology to Computation: Proceedings of the Smalefest* (New York: Springer, 1993); and Steve Batterson, *Stephen Smale: The Mathematician Who Broke the Dimension Barrier* (Providence, R.I.: American Mathematical Society, 2000).

35 Marston Morse, "The Calculus of Variation in the Large," *Collected Papers* (Singapore: World Scientific, 1987), 1:423; *The Calculus of Variation in the Large* (New York: American Mathematical Society, 1934); and the famous textbook by John Milnor, *Morse Theory* (Princeton, N.J.: Princeton University Press, 1963).

36 See H. Whitney, "Singularities of Mappings in Euclidean Spaces," *Symposium internacional de topología algebraica* (Mexico City: Universidad Nacional Automa de México and UNESCO, 1958), 285–301.

37 H. Whitney, "On Singularities of Mappings of Euclidean Spaces. I. Mappings of the plane into the plane," *Annals of Mathematics* 62 (1955): 374–410; repr. in *Collected Papers* (Berlin: Birkhäuser, 1992), 370–406.

38 Castrigiano and Hayes, *Catastrophe Theory*. See SSM, 29–34; and MMM, 59–77.

39 John Guckenheimer, review of SSM, *Bulletin of the American Mathematical Society* 79 (1973): 878–890; A. Majthay, *Foundations of Catastrophe Theory* (Boston: Pitman, 1985), 1; Michel Demazure, *Catastrophes et bifurcations* (Paris: Ellipse, 1989), 167. The title of this section is a quote from Castrigiano and Hayes, *Catastrophe Theory*, xii.

40 Thom, "Une théorie dynamique," and SSM. For the date, see Thom, "Problèmes rencontrés," 203.

41 Tim Poston and Ian Stewart, *Catastrophe Theory and Its Applications* (London: Pitman, 1978), 2.

42 Two recent books are essentially dedicated to a pedagogical reproduction of this proof: Demazure, *Catastrophes et bifurcations*, and Castrigiano and Hayes, *Catastrophe Theory*. Poston and Stewart present an intermediate-level explanation of the notions that articulate this theorem; see their chapter 7 in Castrigiano and Hayes, *Catastrophe Theory*, 99–122.

43 E. C. Zeeman's most famous articles on catastrophe theory were gathered in his *Catastrophe Theory: Selected Papers, 1972–1977* (Reading, Mass.: Addison-Wesley, 1977). See in particular the Thom-Zeeman debate, 615–650.

44 Poston and Stewart, *Catastrophe Theory*, 7.

45 Thom, *Paraboles et catastrophes*, 98; see also René Thom, "Le statut épistémologique de la théorie des catastrophes," *Morphologie et imaginaire, Circé*, 8/9 (1978): 7–24; repr. in *Apologie du logos* (Paris: Hachette, 1990), 395–410.

46 Waddington, preface to *Towards a Theoretical Biology*, 1.

47 On this institute, see David Aubin, "Un pacte singulier entre mathématiques et industrie: L'enfance chaotique de l'Institute des hautes études scientifiques," *La Recherche*, no. 313 (1998): 98–103; and A. Jackson, "The IHÉS at Forty," *Notices of the American Mathematical Society* 46(3) (1999): 329–337.

48 Thom, *Prédire n'est pas expliquer*, 27. For his early philosophical interest, see ibid., 14; and Jacques Nimier (ed.), *Entretiens avec des mathématiciens (L'heuristique mathématique)* (Villeurbanne: Institut de Recherche en Enseignement des Mathématiques, 1989), 96–97.

49 Thom, "Exposé introductif," 27. He also said, "I never mistook myself for a mathematician"; see *Paraboles et catastrophes*, 29.

50 René Thom, "La stabilité topologique des applications polynomiales," *L'Enseignement mathématique*, 2nd ser., 8 (1962): 24–33; and "Ensembles et morphismes stratifiés," *Bulletin of the American Mathematical Society* 75 (1969): 240–284.

51 Zeeman, *Catastrophe Theory: Selected Papers*, 373.

52 Thom, *Prédire n'est pas expliquer*, 27; my emphasis. About the catastrophe theory approach of caustics, see Michael V. Berry, "Les jeux de lumières dans l'eau," *La Recherche* 9 (1978): 760–768.

53 Thom, *Paraboles et catastrophes*, 45. Gastrulation is the process by which the first internal layer of cells is formed in an animal embryo.

54 Thom, "Exposé introductif," 30; see also Max Delbrück's comment in *Unités biologiques douées de continuité génétique* (Paris: CNRS, 1949), 33–34, trans. in MMM, 29–31. E. C. Zeeman, "The Topology of the Brain and Visual Perception," in M. K. Fort (ed.), *Topology of 3-Manifolds and Related Topics: Proceedings of the University of Georgia Institute 1960–61* (Englewood Cliffs: Prentice-Hall, 1962), 240–256; and "Topology of the Brain," *Mathematics and Computer Science in Biology and Medicine* (London: Her Majesty's Stationery Office, 1965), 277–292.

55 SSM, xxiii. About von Uexküll and Goldstein, see Ann Harrington, *Reenchanted Science: Holism in German Culture from Wilhelm II to Hitler* (Princeton, N.J.: Princeton University Press, 1996).

56 Thom, "Une théorie dynamique," 152; see also MMM, 14.

57 Conrad Hal Waddington, *The Strategy of the Genes: A Discussion of Some Aspects of Theoretical Biology* (London: Allen and Unwin, 1957), 4, 9. On Waddington, see Alan Robertson, "Conrad Hal Waddington," *Biographical Memoirs of Fellows of the Royal Society* 23 (1977): 575–622; and Donna J. Haraway, *Crystals, Fabrics, and Fields: Metaphors of Organicism in Twentieth-Century Developmental Biology* (New Haven: Yale University Press, 1976).

58 Conrad Hal Waddington, "The Basic Ideas of Biology," in Waddington (ed.), *Towards a Theoretical Biology*, 1:9. See also Waddington, *Organisers and Genes* (Cambridge: Cambridge University Press, 1940) and *Principles of Embryology* (London: Allen and Unwin, 1956).

59 Conrad Hal Waddington, "The Theory of Evolution Today," in Arthur Koestler and J. R. Smythies (eds.), *Beyond Reductionism: New Perspectives in the Life Sciences* (London: Hutchinson, 1969), 363.

60 Scott F. Gilbert has examined the source of this idea in his "Epigenetic Landscaping:

Waddington's Use of Cell Fate Bifurcation Diagrams," *Biology and Philosophy* 6 (1991): 135–154. The epigenetic landscape first appeared in *An Introduction to Modern Genetics* (New York: MacMillan, 1939) and was treated extensively in *Organisers and Genes* and *Strategy of the Genes*.

61 Waddington, *Strategy of the Genes*, 29.

62 From the "Greek roots *χrh*, it is necessary, and οδοξ, a route or path." Waddington, *Strategy of the Genes*, 32.

63 See "Correspondence Between Waddington and Thom," in Waddington (ed.), *Towards a Theoretical Biology*, 1:166–179.

64 It is unclear whether Smale or Thom first introduced this concept. According to Robert Williams, "each says the other invented it"; see *From Topology to Computation*, 183. For their first definitions, see *SSM*, 39, and Smale, *Mathematics of Time*, 20. At least once, however, Thom claimed responsibility for the term, while borrowing Smale's definition: "Problèmes rencontrés," 203–204.

65 Thom, "Une théorie dynamique," 158; a translation appears in *MMM*, 19.

66 Waddington, *Organisers and Genes*, quoted in Robertson, "Conrad Hal Waddington," 593.

67 C. H. Waddington, *SSM*, xxi; my emphasis. See also his "Theory of Evolution," 367.

68 René Thom, "A Global Dynamical Scheme for Vertebrate Embryology," *Lectures on Mathematics in the Life Sciences* 5 *(Some Mathematical Questions in Biology IV: Proceedings of the Sixth Symposium on Mathematical Biology)* (Providence, R.I.: American Mathematical Society, 1973), 44.

69 Jacques Monod, *Chance and Necessity* (New York: Knopf, 1971), 81, 88; my emphasis.

70 Ibid., 89, 95.

71 Note, however, that neither Thom nor Monod mentioned the work of the other in their writings. Indeed, Monod wished to counter vague approaches based on "general systems theory"; see ibid., 80.

72 René Thom, "D'un modèle de la science à une science des modèles," *Synthese* (1975): 359–374; my emphasis.

73 Françoise Gail, "De la résistance des biologistes à la théorie des catastrophes," in Jean Petitot (ed.), *Logos et théorie des catastrophes* (Geneva: Patiño, 1988), 269–279. One may note the more ambivalent position defended by François Jacob in "Le modèle linguistique en biologie," *Critique* 30(322) (1974): 197–205; Henri Atlan, *Entre le cristal et la fumée: Essai sur l'organisation du vivant* (Paris: Seuil, 1979), 219–229; and Michael A. B. Deakin, "The Impact of Catastrophe Theory on the Philosophy of Science," *Nature and System* 2 (1980): 177–288.

74 Thom, manuscript for *SSM*, sect. 13.3.C., Fine Library, Princeton University.

75 Thom, "Topologie et signification," *L'Âge de la science* 4 (1968); repr. in *MMM*, 166–191.

76 Thom, "Topologie et signification," in *MMM*, 169; "Topological Models in Biology," in Waddington (ed.), *Towards a Theoretical Biology* (Edinburgh: Edinburgh University Press, 1970), 3:89–116, 103; repr. from *Topology* 8 (1969): 313–335.

77 René Thom, "Topologie et linguistique," in André Heafliger and R. Narasimhan (eds.), *Essays on Topology and Related Topics (Dedicated to G. de Rham)* (Berlin: Springer, 1970), 148–177; repr. in*MMM*, 192–213.

78 Louis Tesnière, *Élements de syntaxe structurale* (Paris: Klincksieck, 1965).

79 René Thom, *Semiophysics: A Sketch*, trans. Vendla Meyer (Redwood City: Addison-Wesley, 1966), viii.

80 Thom, "Topologie et linguistique," in *MMM*, 197. See a figure of sixteen archetypal types in *SSM*, 307.

81 See Aubin, "Withering Immortality"; Michel Serres, *Hermès V. Le Passage du Nord-Ouest* (Paris: Seuil, 1980), 99; and Jean-François Lyotard, *The Postmodern Condition: A Report on Knowledge*, trans. G. Bennington and B. Massumi (Minneapolis: University of Minnesota Press, 1984), 60.

82 René Thom, "Structuralism and Biology," in Waddington (ed.), *Towards a Theoretical Biology* (Edinburgh: Edinburgh University Press, 1972), 4:68, 70; this article also appeared in the first French edition of *MMM* (1974) but was absent from later editions.

83 For a contemporary attempt at articulating a multidimensional structuralism, see a book by one of Thom's followers, Paul Scheurer, *Révolutions de la science et permanence du réel* (Paris: Presses Universitaires France, 1979).

84 Thom calls this second approach "l'approche structurale." We must note a difference between French qualifiers: *structurel* (as in "stabilité structurelle," simply the translation of an English phrase) refers to actual structures, while *structural* refers to structures as syntax, which can be realized in several instances of actual structures; see Jean-Marie Auzias, *Clefs pour le structuralisme* (Paris: Seghers, 1967), 18.

85 Thom, "La science malgré tout . . . ," 6; "La linguistique," 239.

86 Concerning Piaget, see Aubin, "Withering Immortality."

87 Thom, "La linguistique," 240.

88 "Expliquer du visible compliqué par de l'invisible simple," introduction to J. Perrin, *Les Atomes* (Paris: Félix Alcan, 1913).

89 Thom, "Topologie et signification," in *MMM*, 174.

90 Thom, *SSM*, 159.

91 Thom, "Généralités sur les morphologies: la description," in French edition of *MMM* (1974), 9, but absent from later editions.

92 Note that there is nothing absolute about the relation between the study of forms and nonreductionism; see, for example, Norma E. Emerton, *The Scientific Reinterpretation of Form* (Ithaca: Cornell University Press, 1984), a historical study that focuses on the struggle to find molecular accounts of crystal shapes.

93 D'Arcy Wentworth Thompson, *On Growth and Form* (1916), abridged ed. by John Tyler Bonner (Cambridge: Cambridge University Press, 1961), 10; my emphasis. Note that Thom used this quotation as an epigraph to his introduction in *SSM*, 1, but dropped the last part where Thompson confidently asserts the success of physics.

94 *SSM*, 9. The allusion to accelerators was added for the second French edition.

95 Benoît B. Mandelbrot, "Towards a Second Stage of Indeterminism in Science," *Interdisciplinary Science Review* 12 (1987): 117.

96 Heraclitus, *Fragments*, trans. T. M. Robinson (Toronto: University of Toronto Press, 1987), fragment 17.

97 "Whatsoever things are objects of sight, hearing, and experience, these things I hold in higher esteem." Heraclitus, *Fragments*, fragment 55.

98 Thom, "Topologie et signification," in *MMM*, 174–175.

99 M. Conche, in Heraclitus, *Fragments* (Paris: Presses Universitaires de France, 1986), 65.

100 Giorgio Israel, *La Mathématisation du réel* (Paris: Seuil, 1996), 198.

101 Thom, manuscript for *SSM*, 13–14, Fine Library, Princeton University; cf. *SSM*, 8–10.

102 Thom, "Une théorie dynamique," 153; *MMM*, 14.

103 Thom, *SSM*, 9; Thompson, *On Growth and Form*, 72–73.

104 Thom, "Modern Mathematics," 697.

105 Thomas S. Kuhn, "Postscript–1969," *The Structure of Scientific Revolutions*, 2d ed. (Chicago: The University of Chicago Press, 1970), 185.

106 Thom, *SSM*, 159. For Rutherford's quote, see p. 4, for example.

107 Thom, "La science malgré tout . . . ," 9–10.

108 Thom, *SSM*, 5; and "Topologie et signification," in *MMM*, 170; italics in the original text.

109 Thom, "Une théorie dynamique," in *MMM*, 21.

110 Thom, "Le découpage qualitatif de la réalité . . . en grandes disciplines: Physique, Chimie, Biologie," in *SSM* (1972), 323. The English translation of *SSM*, 322, misses Thom's point here.

111 Thom, *SSM*, 322; and "Une théorie dynamique," *MMM*, 22.

112 Thom, *Prédire n'est pas expliquer*, 45–46.

113 David Ruelle and Floris Takens, "On the Nature of Turbulence," *Communications in Mathematical Physics* 20 (1971): 167–192; and their "Note" in ibid. 23 (1971): 343–344.

Coping with complexity in technology