Towards the mathematics of quantum field theory

Master course description

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The aim of this course is to gradually introduce master students in mathematics to the mathematical models underlying modern particle physics experiments: quantum field theory. Our aim is to define very precisely mathematically all the ingredients necessary to understand the construction of numbers from models of particle physics, numbers that can be compared to the experiments.

The main reference for the course is a collection of (evolving) notes [Pau09] for a master course prepared in the workshop on mathematical physics at Jussieu's mathematical institute. These are actually based on the following non-exhaustive list of books, whose content will be very partially touched: Derdzinski's book [Der92] on the classical version of the standard model, Folland's book [Fol08] on quantum field theory, Cartier and DeWitt-Morette's book [CDM06] on functional integration, von Neumann's book [vN96] and Takhtajan's book [Tak08] on mathematics of quantum mechanics, Connes and Marcolli's book [CM08] on renormalization, Vinogradov's book [Vin01], Vinogradov-Krasilshchik's book [BCD+99] and Beilinson-Drinfeld's book [BD04] on cohomological approach to variational problems and DeWitt's book [DeW03] on the global approach to quantum field theory.

The plan of the beginning of the course will be:

1. Differential varieties, vector bundles, sheaves.
2. What is a lagrangian: general definition of variational problems.
3. Physical examples: general relativity and electromagnetism.
4. Geometric methods in partial differential equations: jets bundles.
5. Functional derivarites: cohomological methods in variational theory.
6. Representation and structure theory of algebraic reductive groups.
7. Classical matter and interaction particles from the mathematical viewpoint.

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8. Yang-Mills gauge theories.
9. Quantum mechanics following von Neumann.
10. Functional integral quantization.
11. Perturbative expansions.
12. Brief overview of the renormalization procedure.

If time permits, we could also treat the following specialized matters:

1. The BRST/BV method for quantizing gauge theories.
2. Renormalization following Connes-Kreimer-Marcolli.
3. The BRST/BV method on covariant phase space.

Prerequisites:

- basic differential geometry (vector bundle, tangent bundle). This will be explained rapidly at the beginning of the course.
- basics on Hilbert spaces over $\mathbb{C}$.
- a bit of commutative algebra.

References

[BCD+99] A. V. Bocharov, V. N. Chetverikov, S. V. Duzhin, N. G. Khor′kova, I. S. Krasil′shchik, A. V. Samokhin, Yu. N. Torkhov, A. M. Verbovetsky, and A. M. Vinogradov. *Symmetries and conservation laws for differential equations of mathematical physics*, volume 182 of *Translations of Mathematical Monographs*. American Mathematical Society, Providence, RI, 1999. Edited and with a preface by Krasil′shchik and Vinogradov, Translated from the 1997 Russian original by Verbovetsky [A. M. Verbovetskiĭ] and Krasil′shchik.

[BD04] Alexander Beilinson and Vladimir Drinfeld. *Chiral algebras*, volume 51 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 2004.

[CDM06] P. Cartier and C. DeWitt-Morette. *Functional integration: action and symmetries*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Cambridge, 2006. Appendix D contributed by Alexander Wurm.

[CM08] Alain Connes and Matilde Marcolli. *Noncommutative geometry, quantum fields and motives*, volume 55 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 2008.

[Der92] Andrzej Derdziński. *Geometry of the standard model of elementary particles.* Texts and Monographs in Physics. Springer-Verlag, Berlin, 1992.

[DeW03] Bryce DeWitt. *The global approach to quantum field theory. Vol. 1, 2*, volume 114 of *International Series of Monographs on Physics.* The Clarendon Press Oxford University Press, New York, 2003.

[Fol08] Gerald B. Folland. *Quantum field theory: a tourist guide for mathematicians*, volume 149 of *Mathematical Surveys and Monographs.* American Mathematical Society, Providence, RI, 2008.

[Pau09] F Paugam. Les mathématiques de la physique moderne, 2009.

[Tak08] Leon A. Takhtajan. *Quantum mechanics for mathematicians*, volume 95 of *Graduate Studies in Mathematics.* American Mathematical Society, Providence, RI, 2008.

[Vin01] A. M. Vinogradov. *Cohomological analysis of partial differential equations and secondary calculus*, volume 204 of *Translations of Mathematical Monographs.* American Mathematical Society, Providence, RI, 2001. Translated from the Russian manuscript by Joseph Krasil′shchik.

[vN96] John von Neumann. *Mathematical foundations of quantum mechanics.* Princeton Landmarks in Mathematics. Princeton University Press, Princeton, NJ, 1996. Translated from the German and with a preface by Robert T. Beyer, Twelfth printing, Princeton Paperbacks.