Selfish Mining in Ethereum

by Cyril Grunspan

École Supérieure Ingénieurs Léonard de Vinci
12 Avenue Léonard de Vinci, 92400 Courbevoie

De Vinci Research Center

Email: `cyril.grunspan@devinci.fr`
Web: `cyrilgrunspan.fr`

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Joint work with Ricardo Pérez-Marco

Talk based on the following articles

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Online mining simulators for Bitcoin exist and confirm our study

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A miner should never mine secretly and never withholds his blocks

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Old question in the Bitcoin community

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4. If S is first to validate a block but then H mines one block before S validates a second one, S broadcasts immediately her secret block. A competition follows. After resolution of this competition, S goes back to 1 (end of a **cycle**).

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6. When the advance of S reduces to 1, S broadcasts her entire fork (end of a **cycle**).
7. (optional **for Bitcoin**) If the advance of S is greater than 2, then each time H mines a block, S broadcasts immediately the part of her fork sharing the same height as the official blockchain

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Strategy with last optional point (7).

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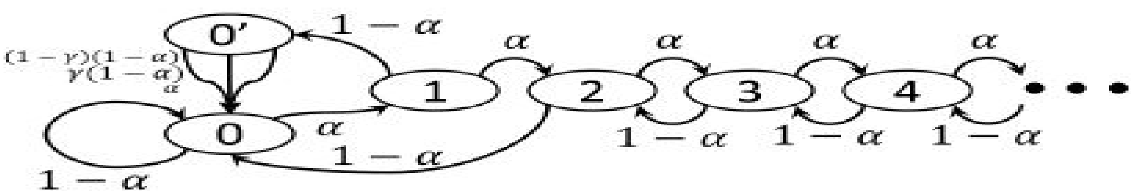

Fig. 1: State machine with transition frequencies.

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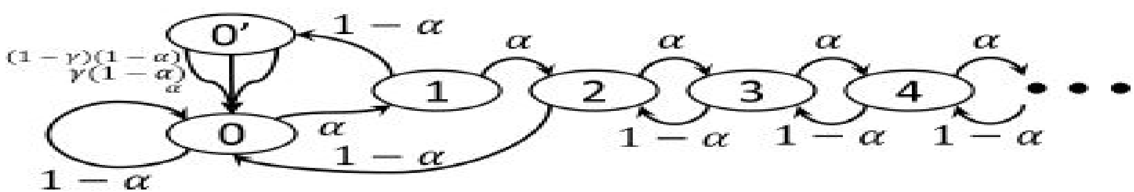

Fig. 1: State machine with transition frequencies.

Each transition gives a reward π for the honest miners and π' for the attacker. These are rewards that the honest miners or the selfish miner will **eventually** earn (possibly not immediatly).

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Definition 4. *Let q' be the mean number of blocks mined by the attacker in the blockchain.*

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Lemma 8. *We have $q' = \frac{\mathbb{E}[\pi']}{\mathbb{E}[\pi] + \mathbb{E}[\pi']}$ where the probability here is the stationary probability.*

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Definition 10. *Let q' be the mean number of blocks mined by the attacker in the blockchain.*

Lemma 11. *We have $q' = \frac{\mathbb{E}[\pi']}{\mathbb{E}[\pi] + \mathbb{E}[\pi']}$ where the probability here is the stationary probability.*

Proof. Strong law of numbers ($\mathbb{E}[\pi] < +\infty, \mathbb{E}[\pi'] < +\infty$):

$$q' = \lim_{n\to\infty} \frac{\pi'_1 + \cdot \quad + \pi'_n}{\pi_1 + \cdot \quad + \pi_n + \pi'_1 + \cdot \quad + \pi'_n} = \lim_{n\to\infty} \frac{\frac{\pi'_1 + \cdot \quad + \pi'_n}{n}}{\frac{\pi_1 + \cdot \quad + \pi_n}{n} + \frac{\pi'_1 + \cdot \quad + \pi'_n}{n}}$$

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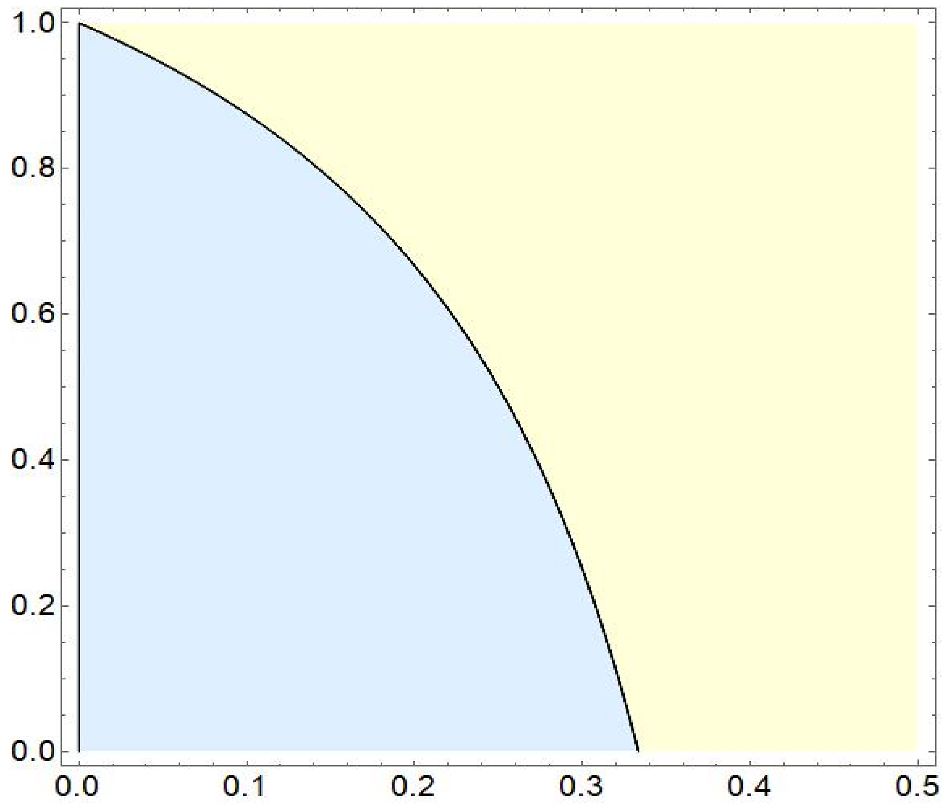
Theorem 15. *We have $q' = \frac{[(p-q)(1+p\,q) + p\,q]\,q - (p-q)\,p^2\,q\,(1-\gamma)}{p\,q^2 + p - q}$*



HM (blue) and SM (yellow) . X-axis: q, Y-axis: γ

Figure 1.

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Controversy: Reality of Selfish Mining?

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Controversy: Reality of Selfish Mining?

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Because none considered the **good objective function** to decide between two strategies

52 Profit and Loss per unit of time

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Time considerations

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Quantity of interest: **profit and loss per unit of time**

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Definition 19. *For any activity with duration time T, we set:*

$$\begin{aligned} \mathrm{PnL} &= R - C \\ \mathrm{PnL}_t &= \frac{R-C}{T} \end{aligned}$$

We set also

$$\mathrm{PnL}_\infty = \lim_{T\to\infty} \frac{R-C}{T}$$

56 Repetitive Games

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Definition 25. *A repetitive strategy is made of repetition of cycles*

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Example 31. A gambler plays repeatedly to a game such as "Head and Tail"

59 Repetitive Games

Definition 35. *A repetitive strategy is made of repetition of cycles*

Example 36. A gambler plays repeatedly to a game such as "Head and Tail"

Definition 37. *We denote by R (resp. C, T) the revenue (resp. cost, duration time) per cycle. The revenue ratio Γ and the cost ratio Υ of an integrable strategy are $\Gamma = \frac{\mathbb{E}[R]}{\mathbb{E}[T]}$ and $\Upsilon = \frac{\mathbb{E}[C]}{\mathbb{E}[T]}$.*

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Theorem 43. *For an integrable repetitive strategy, we have $\mathrm{PnL}_\infty = \frac{\mathbb{E}[R] - \mathbb{E}[C]}{\mathbb{E}[T]}$.*

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Theorem 48. *For an integrable repetitive strategy, we have $\mathrm{PnL}_\infty = \frac{\mathbb{E}[R] - \mathbb{E}[C]}{\mathbb{E}[T]}$.*

Theorem 49. *Let ξ and ξ' be two strategy ξ' sharing the **same cost per unit of time** i.e., $\Upsilon(\xi) = \Upsilon(\xi')$. Then, ξ is less profitable than ξ' if and only if $\Gamma(\xi) < \Gamma(\xi')$*

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- So, we can assume that the coinbase includes fees: $b \leftarrow b + \mathbb{E}[f]$
- The relation $\xi \prec \xi'$ is independent with the amount of fees per block.
- **The revenue ratio is the good notion to decide between two mining strategies**

69 Bitcoin's stability theorem

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Theorem 51. *Without a difficulty adjustment, the best strategy is the honest one.*

71 Bitcoin's stability theorem

Theorem 52. *Without a difficulty adjustment, the best strategy is the honest one.*

Proof. For $t \in \mathbb{R}_+$, we denote by $\boldsymbol{N}(t)$ resp. $\boldsymbol{N}'(t)$) the number of blocks validated by the honest miners (resp. attacker) between 0 and t.

Without a difficulty adjustment, $\boldsymbol{N}(t)$, (resp. $\boldsymbol{N}'(t)$) is a true Poisson process with parameter $\alpha = \frac{p}{\tau_0}$ (resp. $\alpha' = \frac{q}{\tau_0}$) and $R(t) \leqslant \boldsymbol{N}'(t)$.

For any integrable stopping time τ, $\boldsymbol{N}(\tau) - \alpha\tau$ (resp. $\boldsymbol{N}'(\tau) - \alpha\tau$) is a martingale.

Then, we apply Doob's theorem. We get $\frac{\mathbb{E}[R(\tau)]}{\mathbb{E}[\tau]} \leqslant q\frac{b}{\tau_0} = \Gamma(\mathrm{HM})$. □

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So, the problem lies in the difficulty adjustment formula

73 Bitcoin difficulty adjustment

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The difficulty adjustment in Bitcoin today is $D_{\text{new}} = D_{\text{old}} \cdot \frac{n_0 \tau_0}{\boldsymbol{S}_{n_0}}$ where $\boldsymbol{S}_{\boldsymbol{n_0}}$ is the time used to mine $n_0 = 2016$ blocks.

75 Bitcoin difficulty adjustment

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Note 56. In reality, due to a well known bug, it is $D_{\text{new}} = D_{\text{old}} \cdot \frac{n_0 \tau_0}{\boldsymbol{S}_{n_0 - 1}}$. So, if there is no attacker and the difficulty parameter remains constant, the exact mean interblock time τ in Bitcoin is given by ($\frac{1}{\boldsymbol{S}_{n_0 - 1}}$ follows an inverse Gamma distribution):

$$1 = \frac{n_0 \tau_0}{(n_0 - 2)\tau}$$

i.e., $\tau = \tau_0 + \frac{2}{n_0 - 2} \tau_0 > \tau_0$ (inverse Gamma distribution)

76 Analysis of the problem

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BIP proposal

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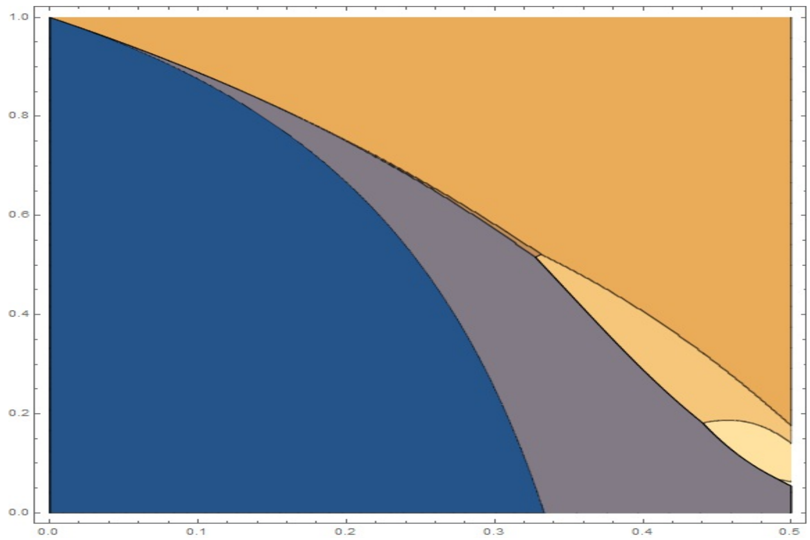

Figure 2. Comparing profitabilities of HM, SM, LSM, EFSM, A-TSM

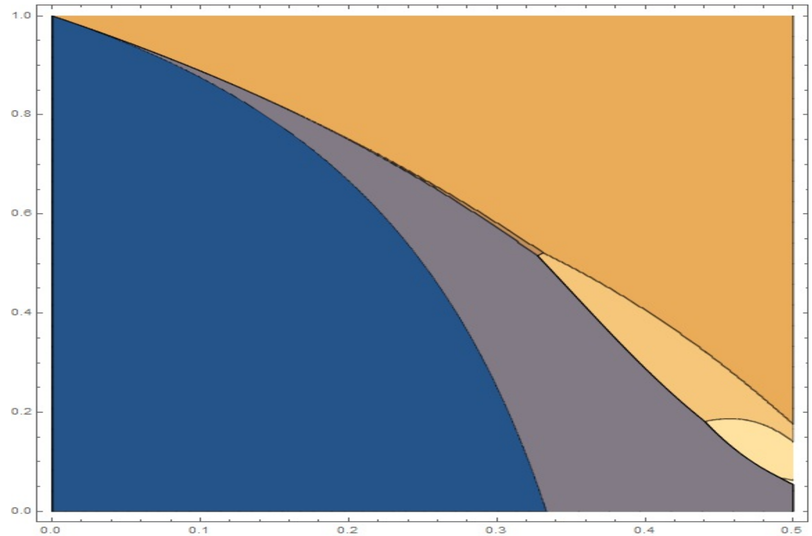

Figure 3. Comparing profitabilities of HM, SM, LSM, EFSM, A-TSM

Optimal strategy obtained by Zohar&al using a **black box solver** of Markov Decision Process

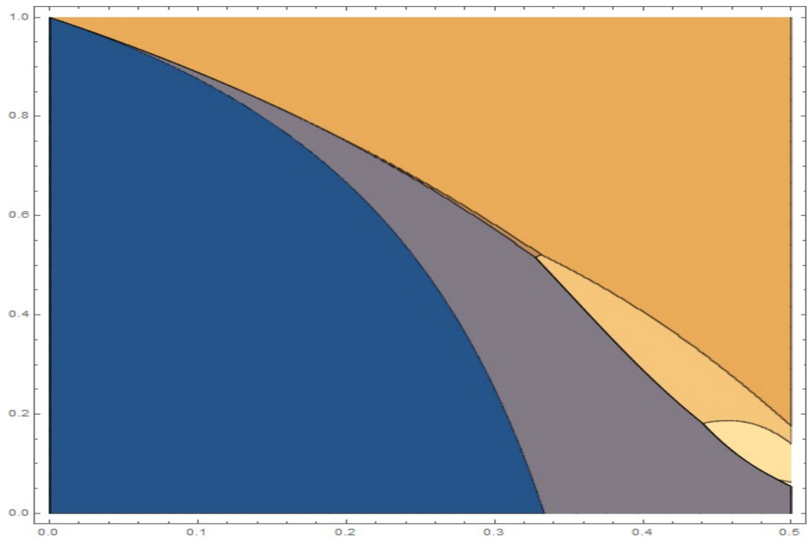

Figure 4. Comparing profitabilities of HM, SM, LSM, EFSM, A-TSM

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Analogous general study missing for Ethereum

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Definition 71. *A Dyck word of length n built on $\{S, H\}$ is a string of S and H containing n S and n H and such that no initial segment of the string has more H's than S's. We denote by $\mathcal{D}_n$ the set of such words and by $\mathcal{D}$ the space of all Dyck words.*

Theorem 72. *The attack cycles of the selfish mining strategies are H, SHS, SHH and SSwH with* $w \in \mathcal{D}$.

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Corollary 75. *We have* $\mathbb{P}[L=1] = p, \mathbb{P}[L=2] = p + p\,q^2$ *and* $\mathbb{P}[L=n] = p\,q^2\,(p\,q)^{n-2}\,C_{n-2}$ *for* $n > 2$ *with* $C_k = \frac{1}{k+1}\binom{2k}{k} = k$*-th Catalan number.*

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We use:

$$\sum_{n\geqslant 0} p\,(p\,q)^n\,C_n = 1$$

$$\sum_{n\geqslant 0} p\,(p\,q)^n\,C_n = \frac{q}{p-q}$$

102 Ethereum network

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Interblock times τ_E reduced: between 13 and 14 sec today

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More or less block propagation time

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Incentives: uncle rewards and inclusion rewards

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Uncle reward $K_u(d) = \frac{8-d}{8} \mathbf{1}_{d \leqslant n_1} b$ with $b = 2$ ETH (coinbase)

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Inclusion reward $K_n(d) = \pi b$ with $\pi = \frac{1}{32}$

117 Main differences with Bitcoin

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A different reward system

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The difficulty adjustment formula in Ethereum is more robust than the difficulty adjustment formula in Bitcoin.

125 Selfish Mining in Ethereum

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Also the attacker can decide to ignore all uncles. She can also signal some uncles...

131 Short Bibliography

132 Short Bibliography

Quite recent topic

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Second article: state machine approach which leads to a quite complicated formula involving a double infinite sum for the long-term apparent hashrate

139 Some definitions

140 Some definitions

Definition 98. *Let* ω *be a cycle. We denote by* $U(\omega)$ *(resp.* $U_S(\omega), U_H(\omega)$*) the number of uncles created during the cycle* ω *which are refered by nephew blocks (resp. nephew blocks mined by the selfish miner, nephew blocks mined by the honest miners) in the cycle* ω *or in a latter cycle.*

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Definition 103. *We denote by R (resp. R_s, R_u, R_n) the revenue (resp. revenue coming from static blocks, uncle rewards, inclusion rewards) of a miner per cycle.*

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Note 108. We have: $R = R_s + R_u + R_n$ and R_s does not depend on the particular strategy.

Lemma 109. *Whatever the selfish mining strategy is, we get $\mathbb{E}[R_u] = p^2\, q\, (1-\gamma)\, K_u(1)$ with $K_u(1) = \frac{7}{8} b$ currently on Ethereum and $\mathbb{E}[R_s] = \mathbb{E}[L] b$ with $\mathbb{E}[L] = 1 + \frac{p^2\, q}{p-q}$.*

143 Selfish mining strategies

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We consider three different selfish mining strategies:

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Strategy 1 maximizes $\mathbb{E}[U]$ and $\mathbb{E}[R]$.

Strategy 2B minimizes $\mathbb{E}[U]$ and $\mathbb{E}[R]$.

Strategy 2A in the middle...

150 Revenue ratio with the new difficulty adjustment formula

151 Revenue ratio with the new difficulty adjustment formula

The revenue ratio of a strategy (recent DA on Ethreum) is proportional to

$$\begin{aligned}\tilde{\Gamma}_E &= \frac{\mathbb{E}[R]}{\mathbb{E}[L]+\mathbb{E}[U]} \\ &= \frac{\mathbb{E}[R_s]+\mathbb{E}[R_u]+\mathbb{E}[R_n]}{\mathbb{E}[L]+\mathbb{E}[U]}\end{aligned}$$

152 Revenue ratio with the new difficulty adjustment formula

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Only the terms in red depend on the strategy.

153 Revenue ratio with the new difficulty adjustment formula

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Only the terms in red depend on the strategy.

Strategy 1 maximizes the numerator (but also the denominator). Strategy 2B minimizes the denominator (but also the numerator).

154 Revenue ratio with the new difficulty adjustment formula

The revenue ratio of a strategy (recent DA on Ethreum) is proportional to

$$
\begin{aligned}
\tilde{\Gamma}_E &= \frac{\mathbb{E}[R]}{\mathbb{E}[L]+\mathbb{E}[U]} \\
&= \frac{\mathbb{E}[R_s]+\mathbb{E}[R_u]+\mathbb{E}[R_n]}{\mathbb{E}[L]+\mathbb{E}[U]}
\end{aligned}
$$

Only the terms in red depend on the strategy.

Strategy 1 maximizes the numerator (but also the denominator). Strategy 2B minimizes the denominator (but also the numerator).

Theorem 114. *We have:* $\tilde{\Gamma}_E = \tilde{\Gamma}_B \cdot \frac{\mathbb{E}[L]}{\mathbb{E}[L]+\mathbb{E}[U]} + \frac{p^2 q\, K_u(1)}{\mathbb{E}[L]+\mathbb{E}[U]} + \frac{\mathbb{E}[U_S]}{\mathbb{E}[L]+\mathbb{E}[U]}\,\pi$

155 Dyck words, Dyck paths and probability space

156 Dyck words, Dyck paths and probability space

A Dyck word w can be identified with a Dyck path $X : [0, 2n] \longrightarrow \mathbb{N}$ such that $X_0 = 0$ and $X_{n+1} = X_n + 1$ (resp. $X_{n+1} = X_n - 1$) if and only if $w_i = S$ (resp. $w_i = H$).

157 Dyck words, Dyck paths and probability space

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The space $\mathcal{D}$ is a probability space with a probability measure $\bar{\mathbb{P}}$ given by $\bar{\mathbb{P}}[w] = p\,(p\,q)^n$ for $w \in \mathcal{D}_n$. If $w \in \mathcal{D}$, then $\mathbb{P}[\omega = \mathrm{SS}\,w\,H] = q^2\,\bar{\mathbb{P}}[w]$.

158 Dyck words, Dyck paths and probability space

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Dyck paths more appropriated than Dyck words for Ethereum for the following reason.

159 Dyck words, Dyck paths and probability space

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Dyck paths more appropriated than Dyck words for Ethereum for the following reason.

Proposition 119. *Let ω be an attack cycle with $\omega = SSwH$ and $w \in \mathcal{D}$. Let $\mathfrak{b}_i$ be the i-th block validated in ω. If $\mathfrak{b}_i$ is an uncle, then $X_i = X_{i-1} - 1$ and $X_i < n_1 - 2$.*

Proof. We have that $X_i + 2 = h(\mathfrak{f}) - h(\mathfrak{b}_i)$ where $h(\mathfrak{f})$ (resp. $h(\mathfrak{b}_i)$) is the the height of the secret block at the time of the creation of $\mathfrak{b}_i$ (resp. the height of $\mathfrak{b}_i$). $\square$

160 Strategy 1: Maximum Belligerence & refers all (classical case)

161 Strategy 1: Maximum Belligerence & refers all (classical case)

We need to compute $\mathbb{E}[U_S]$ and $\mathbb{E}[U]$.

162 Strategy 1: Maximum Belligerence & refers all (classical case)

We need to compute $\mathbb{E}[U_S]$ and $\mathbb{E}[U]$.

We can precise Proposition 119.

Proposition 126. *Let ω be a cycle with $\omega = SSwH$ and $w \in \mathcal{D}$. Let $\mathfrak{b}_i$ be the i-th block validated in ω. If $X_i < X_{i-1}$ and $X_i < n_1 - 2$ then $\mathfrak{b}_i$ is an uncle with probability γ unless $X_i < n_1 - 2$ and $\mathfrak{b}_i$ is the first block validated by the honest miners.*

163 Strategy 1: Maximum Belligerence & refers all (classical case)

We need to compute $\mathbb{E}[U_S]$ and $\mathbb{E}[U]$.

We can precise Proposition 119.

Proposition 129. *Let ω be a cycle with $\omega = SSwH$ and $w \in \mathcal{D}$. Let $\mathfrak{b}_i$ be the i-th block validated in ω. If $X_i < X_{i-1}$ and $X_i < n_1 - 2$ then $\mathfrak{b}_i$ is an uncle with probability γ unless $X_i < n_1 - 2$ and $\mathfrak{b}_i$ is the first block validated by the honest miners.*

Definition 130. *If ω is a cycle starting with SS, we denote by $H(\omega)$ the number of blocks mined by the honest miners and corresponding to an index i such that $X_i < X_{i-1}$ and $X_i < n_1 - 2$.*

164 Strategy 1: Maximum Belligerence & refers all (classical case)

We need to compute $\mathbb{E}[U_S]$ and $\mathbb{E}[U]$.

We can precise Proposition 119.

Proposition 132. *Let ω be a cycle with $\omega = SSwH$ and $w \in \mathcal{D}$. Let $\mathfrak{b}_i$ be the i-th block validated in ω. If $X_i < X_{i-1}$ and $X_i < n_1 - 2$ then $\mathfrak{b}_i$ is an uncle with probability γ unless $X_i < n_1 - 2$ and $\mathfrak{b}_i$ is the first block validated by the honest miners.*

Definition 133. *If ω is a cycle starting with SS, we denote by $H(\omega)$ the number of blocks mined by the honest miners and corresponding to an index i such that $X_i < X_{i-1}$ and $X_i < n_1 - 2$.*

Proposition 134. *We have:* $\mathbb{E}[H(\omega)|\omega = \mathrm{SS}*] = \frac{p}{p-q}\left(1 - \left(\frac{q}{p}\right)^{n_1-1}\right)$

Proposition 135. *We have:* $\mathbb{E}[U] = q + \frac{q^3\gamma}{p-q} - \frac{p^3}{p-q}\left(\frac{q}{p}\right)^{n_1+1}\gamma - q^{n_1+1}(1-\gamma)$

Proposition 136. *We have:* $\mathbb{E}[U] = q + \frac{q^3\gamma}{p-q} - \frac{p^3}{p-q}\left(\frac{q}{p}\right)^{n_1+1}\gamma - q^{n_1+1}(1-\gamma)$

Proof. We have $U(\{H\}) = 0$ and $U(\omega) = 1$ if $\omega \in \{\mathrm{SHS}, \mathrm{SHH}\}$. Also,

$$\mathbb{E}[U|\omega = \mathrm{SS.}\ \ ] = \mathbb{E}[H(\omega)|\omega = \mathrm{SS}]\gamma + (1-\gamma)(p + p\,q + .\ \ + p\,q^{n_1-2})$$

Indeed, there is a probability γ that a block $\mathfrak{b}_i$ satisfying $X_i = X_{i-1} - 1$ and $X_i < n_1 - 2$ is an uncle except for the first block mined by the honest miners. In this case, the probability is 1. So,

$$\mathbb{E}[U] = p\,q + \left[\frac{p}{p-q}\left(1 - \left(\frac{q}{p}\right)^{n_1-1}\right)\gamma + (1-\gamma)\left(1 - q^{n_1-1}\right)\right]\cdot q^2$$

$\square$

Definition 137. *Let $V(\omega)$ be the number of uncles $\mathfrak{u} \in \omega$ refered by a nephew $\mathfrak{n} \notin \omega$.*

Definition 139. *Let* $V(\omega)$ *be the number of uncles* $\mathfrak{u} \in \omega$ *refered by a nephew* $\mathfrak{n} \notin \omega$.

Lemma 140. *We have:* $\mathbb{E}[V] = \frac{q^2}{p}\,(1 - q^{n_1 - 1})\,\gamma + (1-\gamma)\,p\,q^2\,\frac{1-(p\,q)^{n_1-1}}{1-p\,q}$

Definition 141. *Let $V(\omega)$ be the number of uncles $\mathfrak{u} \in \omega$ refered by a nephew $\mathfrak{n} \notin \omega$.*

Lemma 142. *We have:* $\mathbb{E}[V] = \frac{q^2}{p}(1 - q^{n_1 - 1})\,\gamma + (1 - \gamma)\,p\,q^2\,\frac{1 - (p\,q)^{n_1 - 1}}{1 - p\,q}$

Proof. We have $V(\omega) = 0$ if $\omega \in \{H, \mathrm{SHH}, \mathrm{SHS}\}$. If $\omega = *\mathrm{SHH}.\ \mathrm{H}$ with k H at the end, then only the last $n_1 - 1$ blocks can be uncles signaled by future blocks in the next cycle after ω unless $\omega = \mathrm{SS}.\ \mathrm{SH}.\ \mathrm{H}$ with at most n_1 letters S and $n_1 - 1$ letters H. In that case, the first block validated by the honest miners. So,

$$\mathbb{E}[V] = q^2 \sum_{k \geqslant 1} \inf(k, n_1 - 1)\,p\,q^{k-1}\gamma + (1 - \gamma)\,q \sum_{k=1}^{n_1 - 1} (p\,q)^k$$

Note that $p\,q^{k-1}$ is the probability that a Dyck word ends exactly with $(k - 1)$ H. □

Definition 143. *Let $V(\omega)$ be the number of uncles $\mathfrak{u} \in \omega$ refered by a nephew $\mathfrak{n} \notin \omega$.*

Lemma 144. *We have:* $\mathbb{E}[V] = \frac{q^2}{p} (1 - q^{n_1 - 1})\, \gamma + (1 - \gamma)\, p\, q^2 \frac{1 - (p\, q)^{n_1 - 1}}{1 - p\, q}$

Proof. We have $V(\omega) = 0$ if $\omega \in \{H, \mathrm{SHH}, \mathrm{SHS}\}$. If $\omega = *\mathrm{SHH}.\ \mathrm{H}$ with k H at the end, then only the last $n_1 - 1$ blocks can be uncles signaled by future blocks in the next cycle after ω unless $\omega = \mathrm{SS}.\ \mathrm{SH}.\ \mathrm{H}$ with at most n_1 letters S and $n_1 - 1$ letters H. In that case, the first block validated by the honest miners. So,

$$\mathbb{E}[V] = q^2 \sum_{k \geqslant 1} \inf(k, n_1 - 1)\, p\, q^{k-1} \gamma + (1 - \gamma)\, q \sum_{k=1}^{n_1 - 1} (p\, q)^k$$

Note that $p\, q^{k-1}$ is the probability that a Dyck word ends exactly with $(k-1)$ H. □

Proposition 145. *We have:* $\mathbb{E}[U_h] = p^2 q + (p + (1-\gamma)p^2 q)\,\mathbb{E}[V]$

Proposition 147. *We have:* $\mathbb{E}[U_h] = p^2 q + (p + (1-\gamma)p^2 q)\,\mathbb{E}[V]$

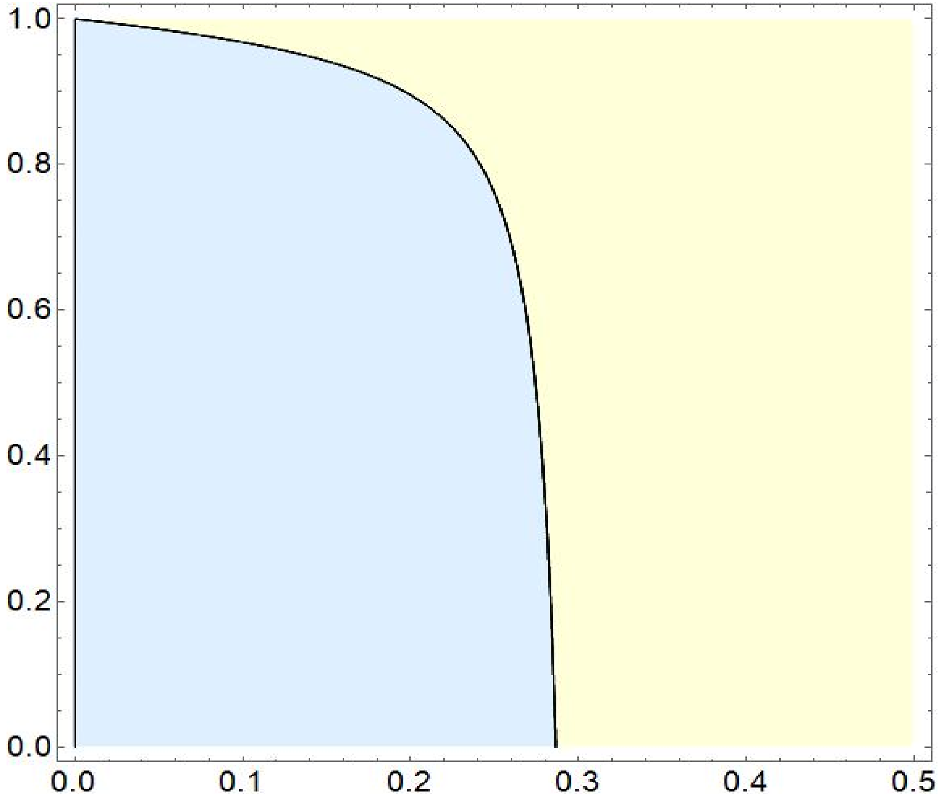
Proof. Let ω be a cycle and let $U_h^{(1)}(\omega)$ (resp. $U_h^{(2)}(\omega)$) be the number of uncles refered by honest nephews only present in ω (resp. not present in ω). Clearly, $\mathbb{E}\big[U_h^{(1)}\big] = p^2 q$. Moreover, the probability that H is the first official block of the next attack cycle is $p + (1-\gamma)p^2 q$. So, $\mathbb{E}\big[U_h^{(2)}\big] = (p + (1-\gamma)p^2 q)\,\mathbb{E}[V]$. □

Proposition 149. *We have:* $\mathbb{E}[U_h] = p^2 q + (p + (1-\gamma)p^2 q)\,\mathbb{E}[V]$

Proof. Let ω be a cycle and let $U_h^{(1)}(\omega)$ (resp. $U_h^{(2)}(\omega)$) be the number of uncles refered by honest nephews only present in ω (resp. not present in ω). Clearly, $\mathbb{E}\big[U_h^{(1)}\big] = p^2 q$. Moreover, the probability that H is the first official block of the next attack cycle is $p + (1-\gamma)p^2 q$. So, $\mathbb{E}\big[U_h^{(2)}\big] = (p + (1-\gamma)p^2 q)\,\mathbb{E}[V]$. □

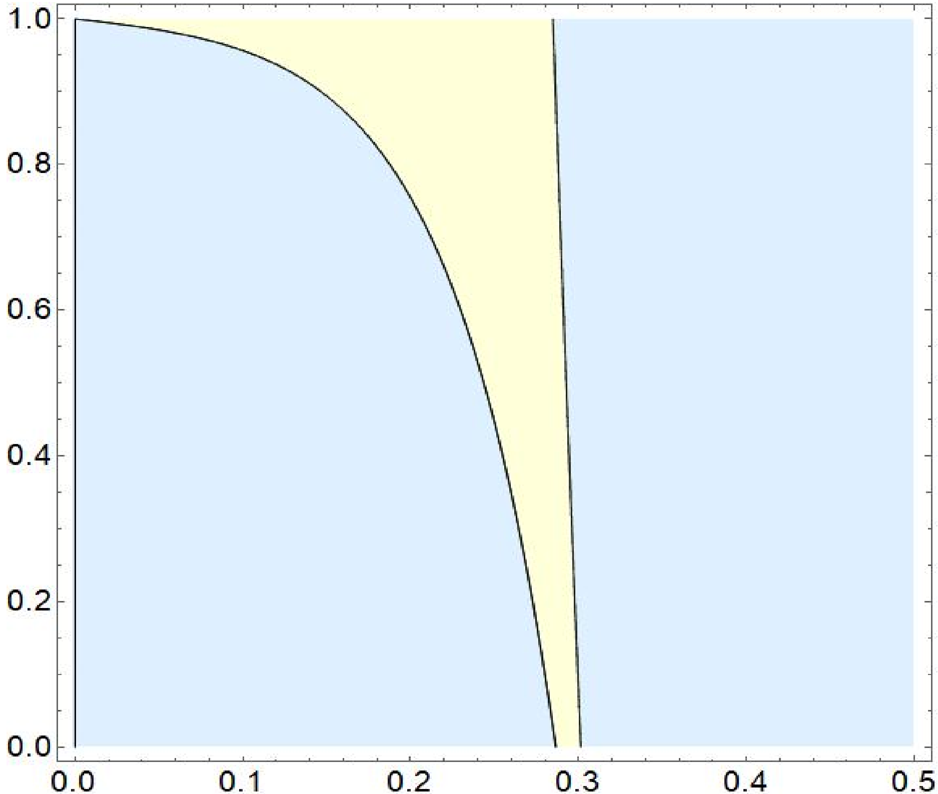
Corollary 150. *We have:*

$$\begin{aligned}\mathbb{E}[U_S] &= q + \frac{q^3\gamma}{p-q} - \frac{p\,q^2}{p-q}\left(\frac{q}{p}\right)^{n_1-1}\gamma - q^{n_1+1}(1-\gamma)\\ &- \left[p^2 q + (p + (1-\gamma)p^2 q)\left(\frac{q^2}{p}(1-q^{n_1-1})\gamma + (1-\gamma)p\,q^2\,\frac{1-(p\,q)^{n_1-1}}{1-p\,q}\right)\right]\end{aligned}$$



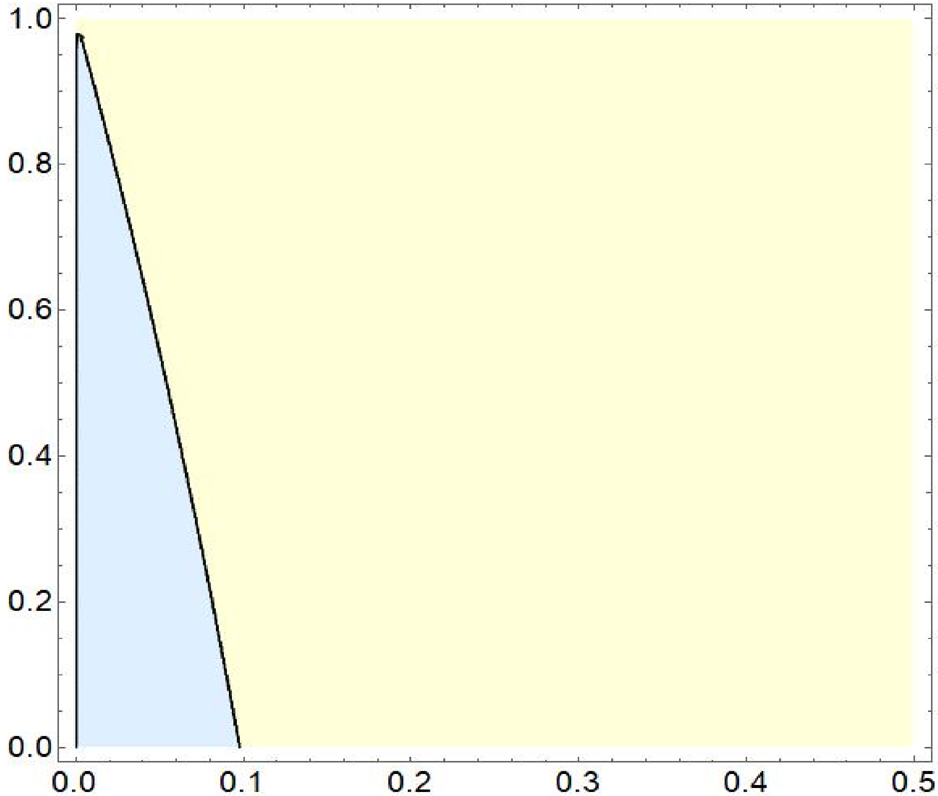
HM (blue) and SM (yellow). X-axis: q, Y-axis: γ

Figure 5.



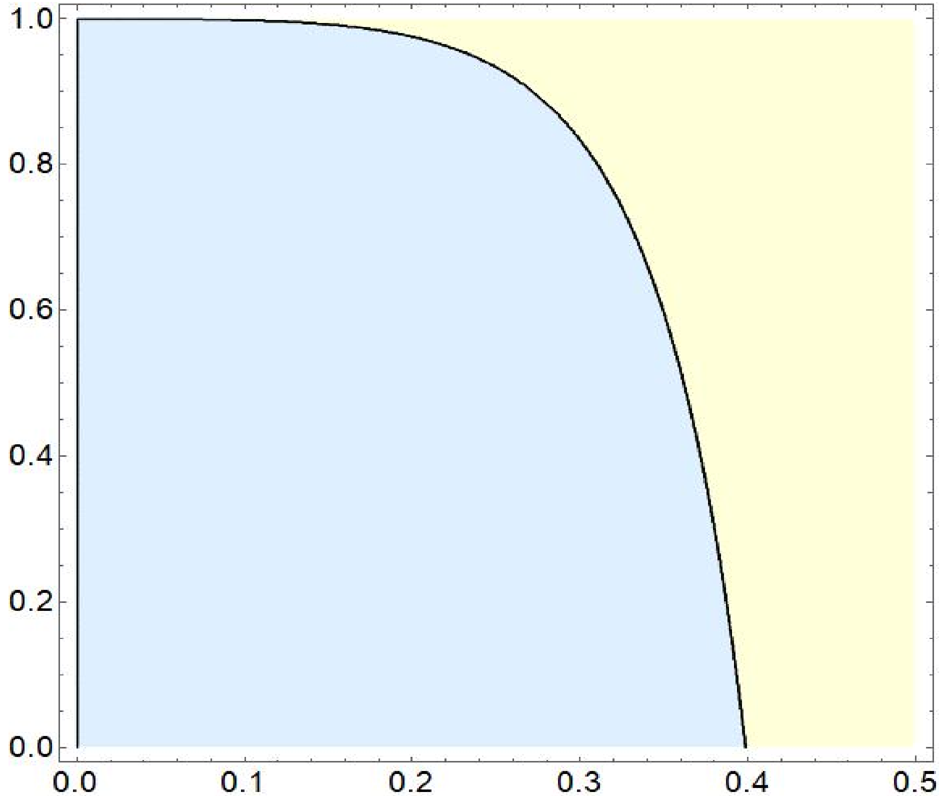
From left to right: HM, SM2A and SM2B

Figure 6.



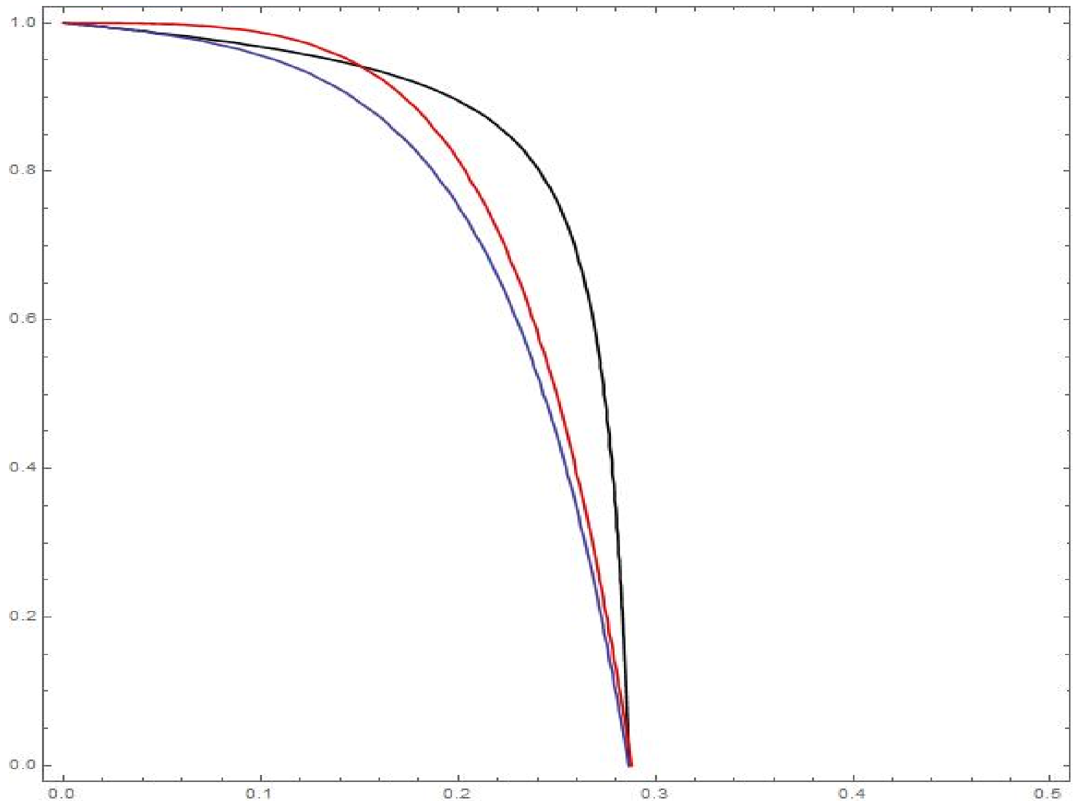
From left to right: HM, SM (old difficulty adjustment)

Figure 7.



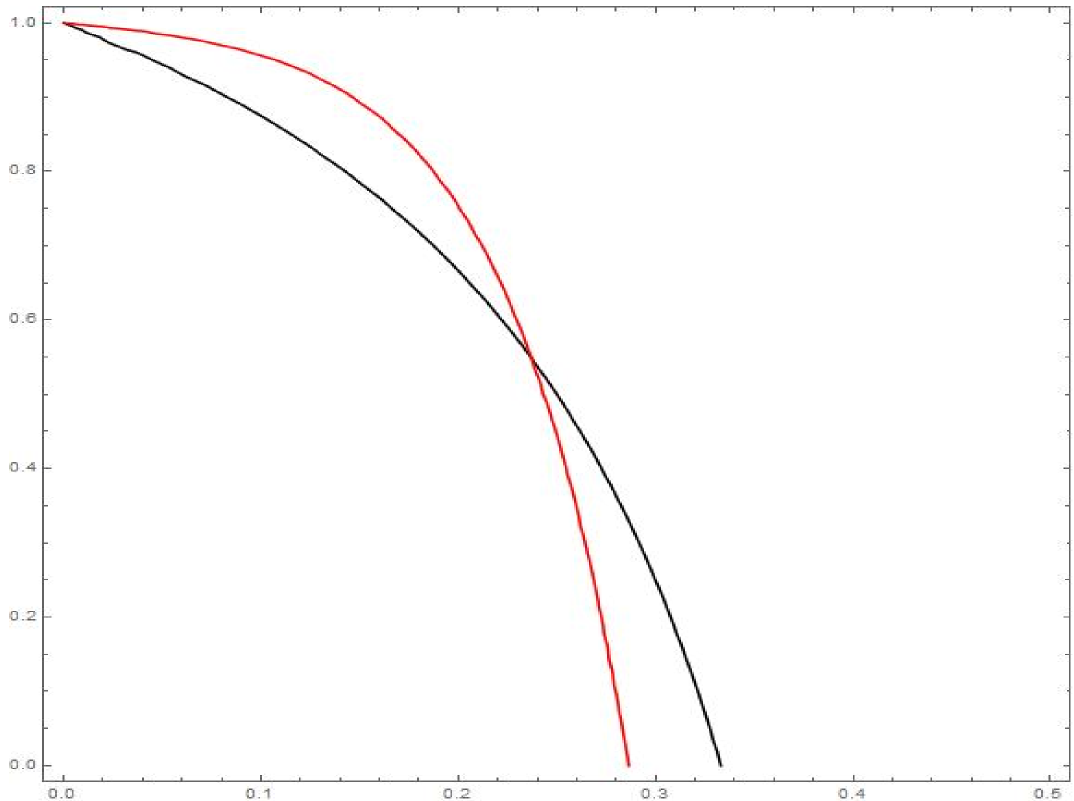
From left to right: HM, SM (possible difficulty adjustment with uncles)

Figure 8.



SM1 (black), SM2A (blue), SM2B (red)

Figure 9.



Thresholds SM Bitcoin (black) & SM2A Ethereum (red)

Figure 10.