On profitability of Selfish Mining (joint work with C. Grunspan)

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"On profitability of Selfish Mining", ArXiv:1805.08281, 5/2018.

On profitability of Selfish Mining

1 Deviant mining strategies

2 On profitability

3 Martingale analysis

4 Lead-Stubborn Mining

5 Equal Fork Stubborn Mining

6 Attack on difficulty adjustment

7 Profitability after a difficulty adjustment

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Bitcoin Stability Conjecture Protocol rules are aligned with self-interest of the network actors.

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- All other things being equal, after 2016 blocks, the diffficulty adjusts down.

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- If $\Delta = 2$ then the SM broadcasts his secret fork.
- If $\Delta \geq 3$ then the SM broadcasts blocks from his secret fork to match the length of the public blockchain.
- Except in the first two cases, the SM keeps working on top of his secret fork.

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Other "Trail Mining" strategies would be discussed elsewhere in the general context of Catch-up Mining (CM).

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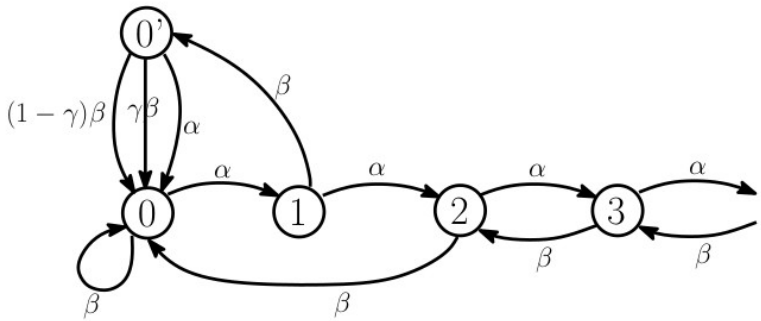

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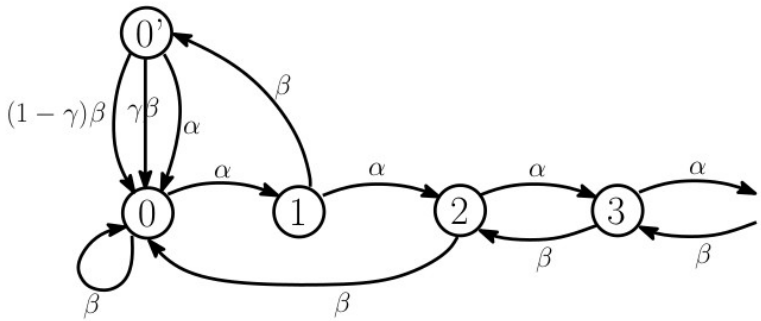

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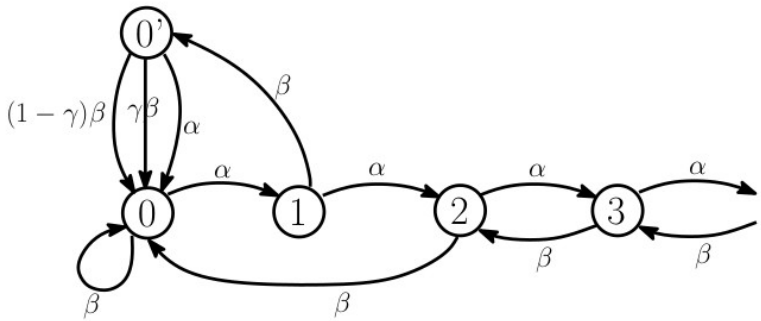

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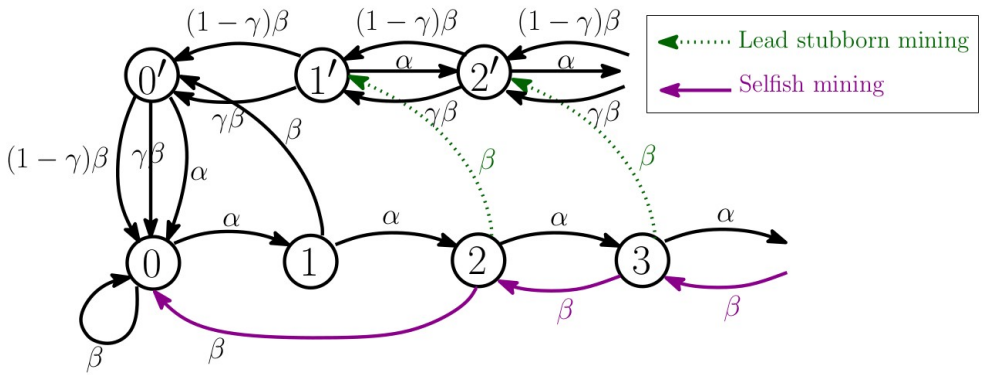

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- The stationnary probability of the Markov model computes the probability of being in a given state in a steady regime.
- The Markov model offers no insight of the duration of the attack cycles nor on the time to reach a steady regime.
- There is no proper analysis of profitability in the literature.

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Theorem (Profitability of integrable games)

$$\mathbb{E}[PnLt] = \frac{\mathbb{E}[R] - \mathbb{E}[C]}{\mathbb{E}[T]} \ .$$

Proof of the Profitability Theorem

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R_i, C_i and T_i values for the i-cycle.

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$$PnLt_n = \frac{\sum_{i=1}^{n} R_i - \sum_{i=1}^{n} C_i}{\sum_{i=1}^{n} T_i} = \frac{\frac{1}{n}\sum_{i=1}^{n} R_i - \frac{1}{n}\sum_{i=1}^{n} C_i}{\frac{1}{n}\sum_{i=1}^{n} T_i}.$$

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$$\mathbb{E}[PnLt] = \lim_{n\to+\infty} PnL_n = \frac{\mathbb{E}[R] - \mathbb{E}[C]}{\mathbb{E}[T]} .$$

□

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Corollary

Let S_1 and S_2 be integrable non-stopping mining strategies. Strategy S_1 is more profitable than strategy S_2 if and only if $P(S_1) \geq P(S_2)$.

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$$\mathbb{P}[N(t) = n] = \frac{(\alpha t)^n}{n!} e^{-\alpha t} \ , \quad \mathbb{P}[N'(t) = n] = \frac{(\alpha' t)^n}{n!} e^{-\alpha' t}$$

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- The revenue ratio of the honest strategy is

$$P(H) = \frac{qb}{\tau_0}$$

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- The combinatorics is involved and the computation of each $T(c)$ involves conditional probabilities and iterated integrals... too complex! We need new tools and ideas...

Martingale Technique

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Definition (Martingale and Stopping Time)

A martingale is a stochastic process $(N(t))_{t\in\mathbb{R}_+}$ with an adapted decreasing filtration $(\Sigma_t)_{t\in\mathbb{R}_+}$ such that $N(t)$ is Σ_t-measurable, and for $t > s$

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N and N' two independent Poisson processes with parameters α' and α with $\alpha' < \alpha$ and $N(0) = N'(0) = 0$. Then, the stopping time

$$\tau = \inf\{t > 0; N(t) = N'(t) + 1\}$$

is finite a.s. and integrable.

Poisson Games

Theorem (Poisson Races)

N and N' two independent Poisson processes with parameters α' and α with $\alpha' < \alpha$ and $N(0) = N'(0) = 0$. Then, the stopping time

$$\tau = \inf\{t > 0; N(t) = N'(t) + 1\}$$

is finite a.s. and integrable. Moreover, we have

$$\mathbb{E}[\tau] = \frac{1}{\alpha - \alpha'}, \quad \mathbb{E}[N'(\tau)] = \frac{\alpha'}{\alpha - \alpha'}, \quad \mathbb{E}[N(\tau)] = \frac{\alpha}{\alpha - \alpha'}.$$

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from where we get

$$\mathbb{E}[\tau] = \frac{1}{\alpha - \alpha'}$$

and the two other formulas. □

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Lemma (Duration of attack cycles)

The duration of attack cycles for selfish mining is given by the stopping time

$$\tau_{SM} = \inf\{t \geq T_1; N(t) = N'(t) - 1 + 2 \cdot \mathbf{1}_{T_1 < T'_1} + 2 \cdot \mathbf{1}_{T'_1 < T_1 < S_2 < S'_2}\}$$

Selfish Mining Revenue Ratio

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$$\mathbb{E}[R(\tau_{SM})] = \frac{(1+pq)(p-q)+pq}{p-q}\, qb - (1-\gamma)p^2 q\, b$$

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and

$$P(SM) = \frac{qb}{\tau_0} - (1-\gamma)\frac{p^2 q(p-q)b}{((1+pq)(p-q)+pq)\,\tau_0} \leq P(H)$$

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Theorem (Optimality of Honest Mining)

For any block withholding strategy S we have

$$P(S) \leq P(H) = \frac{qb}{\tau_0}$$

Lead-Stubborn Mining Stopping Time

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Lemma (Duration of attack cycles)

The duration of attack cycles for Lead-Stubborn mining is given by the stopping time ξ_{LStM}

$$\xi_{LStM} = \tau + \left(T_{N(\tau)+1} \wedge T'_{N(\tau)+1} \right) \cdot \mathbf{1}_{T'_1 \leq T_1}$$

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Note: Same stopping time that for Poisson Games.

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$$P(\xi_{EFStM}) = \frac{qb}{\tau_0} - (p-q)g\frac{b}{\tau_0}$$

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- The apparent hashrate of the attackers becomes $\tilde{q} > q$.
- Depending on q and γ selfish mining strategies can become profitable.

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- This could be cone by propagating orphan headers through the network and include the data in validated blocks.
- Then the new adjustment formula will take the ratio of the difference of first and last timestamp in a n_0 period and divide it by $n_0 + n'$ where n' is the number of orphan blocks.

Apparent hashrates

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Theorem (Apparent hashrates)

$$\tilde{q}(SM) = \frac{((1+pq)(p-q)+pq)q-(1-\gamma)p^2q(p-q)}{p^2q+p-q}$$

$$\tilde{q}(LStM) = q \cdot \frac{p+pq-q^2}{p+pq-q} - \frac{pq(p-q)f(\gamma)}{p+pq-q}$$

$$\tilde{q}(EFStM) = \frac{q}{p} - \left(1-\frac{q}{p}\right) f(\gamma)$$

Expected difficulty adjustments

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Theorem (Expected difficulty adjustments)

$$\mathbb{E}[\delta(SM)] = \frac{p - q + pq(p - q) + pq}{p^2 q + p - q}$$

$$\mathbb{E}[\delta(LStM)] = \frac{p + pq - q^2}{p + pq - q}$$

$$\mathbb{E}[\delta(EFStM)] = \frac{1}{p}$$

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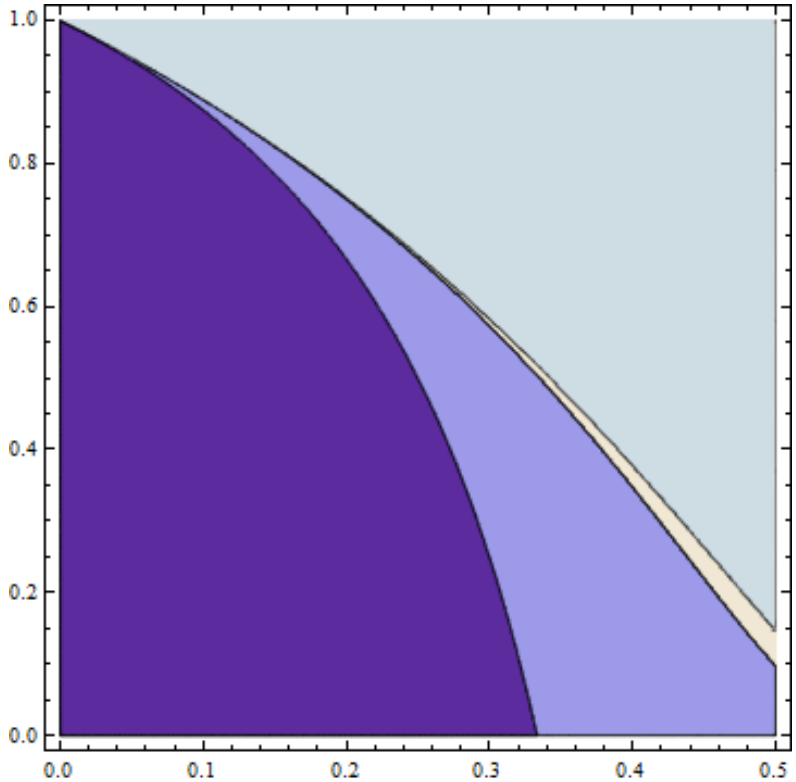

Comparisons with NKMS2016

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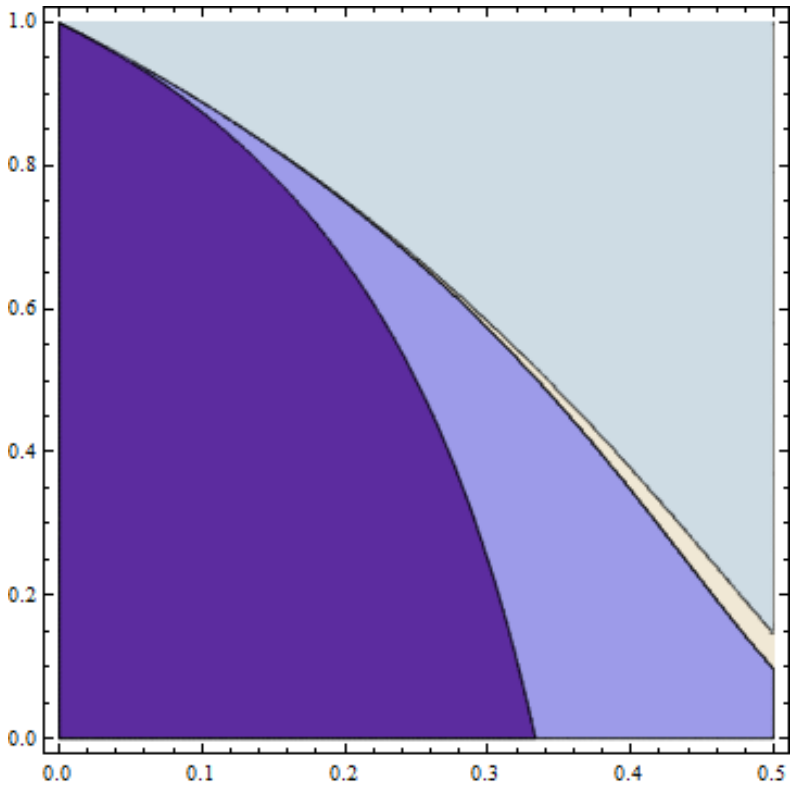

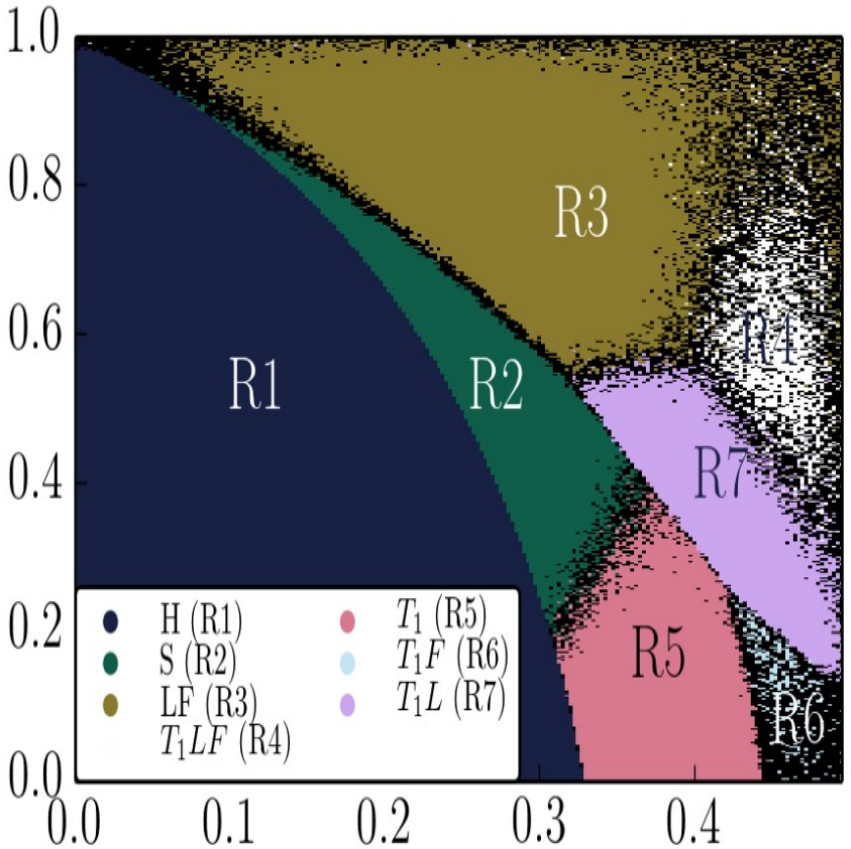

Thank you for your attention!